I. Find an equation for the sphere that has center $(4, -2, \pi)$ and contains the origin.

II. Find parametric equations for the line that is the intersection of the planes $x - 2y - 3z = 1$ and $2x + y + z = 1$.

III. Determine the convergence or divergence of each of these series, using any information or method other than the Limit Comparison Test.

1. $\sum_{n=1}^{\infty} \frac{7 + 7^n}{8 + 8^n}$

2. $\sum_{n=1}^{\infty} \left( \frac{n-1}{n} \right)^n$

IV. Carry out a translation of the form $X = x - x_0, Y = y - y_0, Z = z - z_0$ to put the equation $x^2 - y^2 - 4z^2 = 2x + 16z + 16$ into standard form. (You do not have to draw the graph, but you can if you want.) The graph is a hyperboloid of two sheets. Which traces are ellipses? At what points (in $XYZ$-coordinates) does it meet the $X$-axis?

V. Let $\vec{a} = -4\vec{i} + \vec{j} + 3\vec{k}$ and $\vec{b} = \vec{i} + \vec{j} - \vec{k}$.

1. Calculate the scalar projection of $\vec{a}$ onto $\vec{b}$.

2. Calculate the vector projection of $\vec{a}$ onto $\vec{b}$.

VI. Give an algebraic verification that $\| \vec{a} + \vec{b} \|^2 + \| \vec{a} - \vec{b} \|^2 = 2 \| \vec{a} \|^2 + 2 \| \vec{b} \|^2$, but not by doing a lengthy calculation involving $\vec{i}, \vec{j}$, and $\vec{k}$.

VII. Give examples of the following:

1. Vectors $\vec{a}, \vec{b},$ and $\vec{c}$ for which $(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$.

2. Nonzero vectors $\vec{a}, \vec{b},$ and $\vec{c}$ for which $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ but $\vec{b} \neq \vec{c}$.

VIII. Here is a fact: Let $\vec{u}$ be any unit vector (i.e. $\| \vec{u} \| = 1$). If $\vec{v}$ is any vector, then the length of $\vec{v} \times \vec{u}$ is no more than the length of $\vec{v}$.

1. Give an algebraic explanation for this fact.

2. Give a geometric explanation for this fact.

IX. If you saw a movie of the planes $y \cos(\theta) + z \sin(\theta) = 0$ as $\theta$ went from 0 to $2\pi$, what would they look like? Use both words and picture(s) in your explanation.

X. Derive these formulas expressing rectangular coordinates in terms of spherical coordinates: $x = \rho \sin(\phi) \cos(\theta), y = \rho \sin(\phi) \sin(\theta), z = \rho \cos(\phi)$.

XI. Draw the graph of this equation given in spherical coordinates: $\rho = \phi^3$. 

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XII. Write a possible equation for this saddle surface:

\[ z = \frac{x^2}{y^2} - 1 \]

XIII. In higher dimensions, say dimension \( n \), there are vectors \( \vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n \) that play the roles of \( \vec{i}, \vec{j}, \) and \( \vec{k} \). In particular, \( \vec{e}_i \cdot \vec{e}_j = 0 \) when \( i \neq j \), and \( \vec{e}_i \cdot \vec{e}_i = 1 \) for each \( i \). Verify that if an \( n \)-dimensional vector \( \vec{v} \) equals \( r_1\vec{e}_1 + r_2\vec{e}_2 + \cdots + r_n\vec{e}_n \), then \( r_1 = \vec{v} \cdot \vec{e}_1 \).