I. Determine the convergence or divergence of each of these series, using any information or method other than the Limit Comparison Test. If the series has some negative terms, check for absolute convergence as well.

1. \[ \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n \ln(n)} \]

2. \[ \sum_{n=2}^{\infty} \frac{1}{n \ln(n)} \]

3. \[ \sum_{n=1}^{\infty} \frac{7 + 7^n}{8 + 8^n} \]

4. \[ \sum_{n=1}^{\infty} \left( \frac{n-1}{n} \right)^n \]

II. Find the Maclaurin series for \( \arctan(x) \) by using the fact that \( \arctan(x) = \int \frac{1}{1 + x^2} \, dx \).

III. Analyze the convergence behavior of the power series \( \sum_{n=0}^{\infty} \frac{n}{4^n} (3 - x)^n \). In particular, determine its center, radius of convergence, and for every real number \( x \) determine whether the series converges absolutely, converges conditionally, or diverges.

IV. Using the Maclaurin series of \( \cos(x) \), find the Maclaurin series of the following functions. Make reasonable simplifications.

(i) \( \cos(2x) \)

(ii) \( \sin^2(x) \)

V. Let \( \sum a_n \) be a series with positive terms. Suppose that \( \lim_{n \to \infty} \sqrt[n]{a_n} = L \) with \( 0 < L < 1 \).

1. Let \( r \) be a number with \( L < r < 1 \). Explain (at least informally) why for all sufficiently large values of \( n \), say \( n \geq N \), each \( \sqrt[n]{a_n} < r \).

2. Use the Comparison test to deduce that \( \sum a_n \) converges.

VI. Showing a reasonable amount of detail, use integration by parts to verify that \( \int_a^b \frac{(b - t)^5}{5!} f^{(5)}(t) \, dt = \frac{f^{(6)}(a)}{6!} (b - a)^6 + \int_a^b \frac{(b - t)^6}{6!} f^{(6)}(t) \, dt. \)

VII. Recall that \( \lim_{n \to \infty} \frac{x^n}{n!} = 0 \) (this can be seen, for example, by using the Ratio Test to check that the series \( \sum_{n=0}^{\infty} \frac{x^n}{n!} \) converges for every value of \( x \) and deducing that its terms limit to 0). Use Taylor’s Theorem \( R_n(x) = \int_a^x \frac{(x - t)^n}{n!} f^{(n+1)}(t) \, dt \) to verify that \( e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \) for all \( x > 0 \). (Hint: \( e^t \leq e^x \) for all \( t \) with \( 0 \leq t \leq x \).)
VIII. Recall that \( \lim_{n \to \infty} \frac{x^n}{n!} = 0 \). Use Lagrange’s form \( R_n(x) = \frac{f^{(n+1)}(c)}{(n + 1)!} (x - a)^{n+1} \) to verify that \( e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \) for all \( x > 0 \).

IX. Evaluate \( \int_{0}^{x} e^{-t^2} \, dt \).