I. (7) Analyze the convergence behavior of the power series \( \sum_{n=1}^{\infty} \frac{1}{nb^n} (x - a)^n \), where \( a \) and \( b \) are constants with \( b > 0 \). That is, determine its center, radius of convergence, and for every real number \( x \) determine whether the series converges absolutely, converges conditionally, or diverges.

II. (7) State the Comparison Test, and use it to verify that \( \sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}} \) diverges. (Hint: First verify that \( \lim_{n \to \infty} n^{1/n} = 1 \).)

III. (5) Graph the equation \( r = \cos(\theta/3) \) for \( 0 \leq \theta \leq 6\pi \), first in the \( \theta-r \) plane, then as a polar equation in the \( x-y \) plane.

IV. (7) State the Limit Comparison Test, and use it to verify that \( \sum_{n=1}^{\infty} (\sqrt[n]{2} - 1) \) diverges. (Hints: Use L'Hôpital's Rule to compare it to \( \sum \frac{1}{n} \). You may need the facts that \( \lim_{n \to \infty} \frac{2^{1/n}}{n} = 1 \) and \( \frac{d(a^x)}{dx} = a^x \ln(a) \).)

V. (7) Give examples of the following:
   1. A divergent series whose terms limit to 0.
   3. A geometric series \( \sum_{n=0}^{\infty} r^n \) that converges to \( \pi \).

VI. (6) Give examples of the following:
   1. A power series \( \sum_{n=0}^{\infty} c_n x^n \) that converges only for \( x = 0 \).
   2. A power series \( \sum_{n=0}^{\infty} c_n x^n \) whose radius of convergence is \( \pi \).

VII. (3) Derive these formulas expressing rectangular coordinates in terms of spherical coordinates: \( x = \rho \sin(\phi) \cos(\theta) \), \( y = \rho \sin(\phi) \sin(\theta) \), \( z = \rho \cos(\phi) \).

VIII. (4) In higher dimensions, say dimension \( n \), there are vectors \( \vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n \) that play the roles of \( \vec{i}, \vec{j}, \) and \( \vec{k} \). In particular, \( \vec{e}_i \cdot \vec{e}_j = 0 \) when \( i \neq j \), and \( \vec{e}_i \cdot \vec{e}_i = 1 \) for each \( i \). Verify that if an \( n \)-dimensional vector \( \vec{v} \) equals \( r_1 \vec{e}_1 + r_2 \vec{e}_2 + \cdots + r_n \vec{e}_n \), then \( r_i = \vec{v} \cdot \vec{e}_i \) for each \( i \).

IX. (6) Give examples of the following:
   1. Vectors \( \vec{a}, \vec{b}, \) and \( \vec{c} \) for which \( (\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c}) \).
   2. Nonzero vectors \( \vec{a}, \vec{b}, \) and \( \vec{c} \) for which \( \vec{a} \times \vec{b} = \vec{a} \times \vec{c} \) but \( \vec{b} \neq \vec{c} \).
X. Find an equation for the plane that contains the points (1, 2, 3), (1, 3, 4), and (2, 3, 5).

XI. A point moves according to the vector-valued function \( \mathbf{r}(t) = e^t \mathbf{i} + e^{-t} \mathbf{j} \).

1. Sketch the path of the point, indicating the direction of motion. (Hint: How are \( x \) and \( y \) related?)

2. Calculate the velocity vectors \( \mathbf{r}'(t) \), the speed, and the unit tangent vector \( \mathbf{T}(t) \).

3. Use \( a_T = \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{\| \mathbf{r}'(t) \|} \) and \( a_N = \frac{\| \mathbf{r}'(t) \times \mathbf{r}''(t) \|}{\| \mathbf{r}'(t) \|} \) to calculate the tangential and normal components of the acceleration vector \( \mathbf{a}(t) \).

4. When is the point speeding up? When is it slowing down?

XII. Write the general formula for the Taylor series of a function \( f(x) \) at \( x = a \). Use it to calculate the Taylor series of the function \( f(x) = x^4 \) at \( x = 2 \).

XIII. For the helix \( \mathbf{r}(t) = 2 \sin(t) \mathbf{i} + 3t \mathbf{j} + 2 \cos(t) \mathbf{k} \):

1. Calculate the unit tangent vector \( \mathbf{T}(t) \), and use it to calculate the unit normal \( \mathbf{N}(t) \).

2. Use the formula \( \kappa = \frac{\| \mathbf{T}'(t) \|}{\| \mathbf{r}'(t) \|} \) to calculate the curvature.

3. Use the formula \( \kappa = \frac{d \mathbf{T}}{ds} \) and the Chain Rule to calculate the curvature.

XIV. Bonus Problem: Let \( u = 1 + \frac{x^3}{3!} + \frac{x^6}{6!} + \frac{x^9}{9!} + \cdots, v = x + \frac{x^4}{4!} + \frac{x^7}{7!} + \frac{x^{10}}{10!} + \cdots, \) and \( w = \frac{x^2}{2!} + \frac{x^5}{5!} + \frac{x^8}{8!} + \cdots \). Each of these converges by comparison with the Maclaurin Series of \( e^x \). Show that \( u^3 + v^3 + w^3 - 3uvw = 1 \). (Hint: What is \( u' \)?)