Math 6833 assignments

6. Another one: Set up structures to work with graphs. Write a function to determine whether a graph is connected, or better yet to enumerate its components. Write a function to find a maximal tree in a connected graph.

7. Another one: Encode elements of a free group as lists, for example, $x^3y^{-2}x$ might be `[ ["x", 3 ], ["y", -2 ], ["x", 1 ] ]`. Write a routine that takes a word and returns its reduced form. Adapt it to a version that produces cyclically reduced forms.

8. Here is a starter exercise: Write a GAP routine that uses the Euclidean algorithm to find the greatest common divisor of two integers. Also, look up the built-in GAP routine that does this.

9. These are starter exercises on list manipulation. Read some of the reference manual section on lists, then try some:

    1. Write a routine that takes a list of nonzero integers and returns the rational number that it represents as a continued fraction.
    2. Write a routine that takes a list of integers as its input and returns the maximum element in the list. Also, find the built-in GAP routine that does this.
    3. Write a routine that takes a list as its input and returns another list that is the same as the original except in reversed order. Also, find the built-in GAP routine that does this.
    4. Write a routine that takes two sorted lists of integers (you might want to look at the manual section 21.18 on built-in sorting routines for lists) and merges them into a single ordered list. Of course, you could just concatenate the lists and sort the result again, but write code that merges without requiring a new sort.

10. This is a semi-starter exercise, more a matter of using GAP's built-in functions than programming it: Let G be the group $\langle x, y \mid xyx = yxy, (xy)^3 = 1\rangle$. Let H be the subgroup of G genertd by x and y^2.

    1. Find the Reidemeister-Schreier algorithm in the GAP manual and use GAP to find a presentation of H.
    2. Take the presentation of H that you obtain, and simplify it by hand to show that H is isomorphic to the free product $\mathbb{Z}/2 * \mathbb{Z}$.
    3. Experiment with the presentation simplifier described in section 46.2 of the manual. See how well it works on this example.

11. Read about some of GAP's built-in groups, in section 48 of the reference manual. Using the routine `countGeneratingPairs` in `genPairs.gap`, do some experiments with different groups to get a feel for the probability that a random pair generates, and also for the size of calculation that exceeds GAP's capabilities.