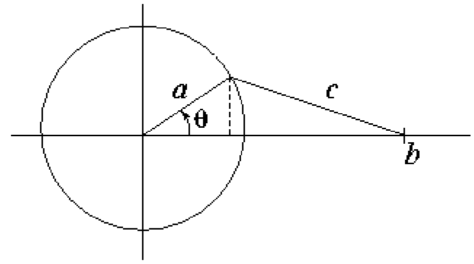
Mathematics 1823-001H Name (please print) ____________________

Examination III

November 21, 2006

Instructions: Give brief, clear answers. It is not expected that most people will be able to answer all the questions, just do what you can in 75 minutes.

I. (6) State the Law of Cosines, and verify it. A helpful figure is shown to the right.



II. (6) The angle of elevation of the sun is decreasing at 0.25 radians per hour. How fast is the length of the shadow of a 100 meter tall tower changing at a time (around 4 p. m.) when the angle of elevation of the sun is $\pi/6$?

III. (15) The Mean Value Theorem states that if a function f is differentiable at all points between a and b, and is continuous at a and b as well, then there exists a c between a and b so that $f(b) - f(a) = f'(c)\,(b - a)$.

1. Find a value that works as the number c in the Mean Value Theorem for the function $x^{2/3}$ on the interval $[0, 8]$.
2. Verify that if $f'(x) \leq 0$ for all x with $a \leq x \leq b$, then $f(b) \leq f(a)$.
3. Verify that if $f'(x) = 0$ for all x in a (connected, but not necessarily closed) interval, then f is constant on the interval.
4. Show that the function $2x - 3 - \sin(x)$ has at most one root between -5 and 5.
5. Show that the function $2x - 3 - \sin(x)$ has at least one root between -5 and 5.

IV. (4) Find all critical points of the function $5t^{2/3} + t^{5/3}$.

V. (4) One of the lines that passes through the point $(2, 0)$ and is tangent to the graph of $y = x^4$ is $y = 0$. Find the other one.

VI. (4) The Extreme Value Theorem says that a continuous function on a closed interval must assume maximum and minimum values.

1. Give an example of a trigonometric function which is continuous on an open interval, and assumes neither a maximum nor a minimum value on the interval.
2. Give an example of a trigonometric function which is continuous on an open interval, and assumes both maximum and minimum values on the interval.

VII. (12) A certain function $f(x)$ has derivative $f'(x) = \frac{x}{x^2+1}$.

1. Determine where $f'(x)$ is positive, and where it is negative.
2. Calculate $f''(x)$. Determine where it is positive, and where it is negative.
3. Where does the minimum value of $f(x)$ occur? Why?
4. Determine where $f(x)$ is concave up, and where it is concave down.
5. Find all inflection points of f.

VIII. (5) Use the definition of rate of change to show that if $f'(a) > 0$, then there exists a $\delta > 0$ so that if $a < a+h < a+\delta$, then $f(a) < f(x)$. Hint: Write $f(x) = f(a) + f'(a)h + E(h)$, where $\lim_{h\to 0} \frac{E(h)}{h} = 0$, and use the observation that $f(a) + f'(a)\,h + E(h) = f(a) + \left(f'(a) + \frac{E(h)}{h}\right) h$.