Instructions: Give brief, clear answers. It is not expected that most people will be able to answer all the questions, just do what you can in 75 minutes.

I.   Find the derivatives of the following functions, using the specified methods.
(12)

    1. $\sqrt{1+2x}$, using the definition $\lim\limits_{h\to x} \dfrac{f(h)-f(x)}{h-x}$.

    2. $1/x^2$, using the definition $\lim\limits_{w\to 0} \dfrac{f(x+w)-f(x)}{w}$.

    3. x^3, using the definition as the best linear approximation.

    4. A function that stretches the line segment $[1/3, 2]$ uniformly to the line segment $[-2/3, 2/3]$, and also reverses its direction, using the concept of derivative as stretch factor.

II.   Find the derivatives of the following functions, using the algebraic rules for sums, products, quotients, and
(9)   the Chain Rule, and the known derivatives of x^r and the trigonometric functions.

    1. $\dfrac{ax+b}{cx+d}$ (where a, b, c, and d are constants).

    2. $\csc^3(x)\sqrt{\sin(x)}$

    3. $L(1/x)$, where $L(x)$ is a function such that $L'(x) = 1/x$.

III.   For the function $f(x) = \sqrt{x}$, obtain a general formula for the nth derivative $f^{(n)}(x)$.
(5)

IV.   Calculate the following limits.
(4)

    1. $\lim\limits_{\theta\to 0} \dfrac{\sin(\theta)}{\theta^2}$

    2. $\lim\limits_{\theta\to 0} \dfrac{\sin(8\theta)}{\sin(7\theta)}$

    3. $\lim\limits_{\theta\to 0} \dfrac{\sin^2(\theta)}{\theta}$

    4. $\lim\limits_{\theta\to 0} \dfrac{\sin(\theta^2)}{\theta^2}$

V.   Calculate $\dfrac{dy}{dx}$ using implicit differentiation if $xy^2 = \cot(xy)$.
(3)

VI.   Calculate $\dfrac{d}{dx}(f(g(x)))\Big|_{x=1}$ if f and g are as given in the table:
(3)

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

VII.   One of the lines that passes through the point $(1,0)$ and is tangent to the graph of $y = x^3$ is $y = 0$. Find
(4)   the other one.

VIII. Use $1 = f \cdot (1/f)$ and the product rule to obtain a formula for $(1/f)'$.
(3)

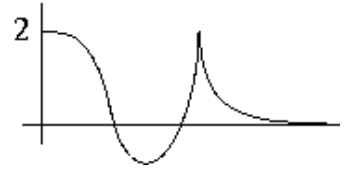**IX.**   Verify that if f is an odd function, then f' is an even function.
(3)

X.   To the right is the graph of a function f. On two separate graphs:
(6)
  (a) Sketch f'.

  (b) Sketch a function F for which $F(0) = 0$ and $F' = f$.

XI.   Challenge Problem: Calculate $\lim\limits_{x \to 1} \dfrac{\sqrt{x} - x^2}{1 - \sqrt{x}}$ by using one of the definitions of the derivative.
(3)