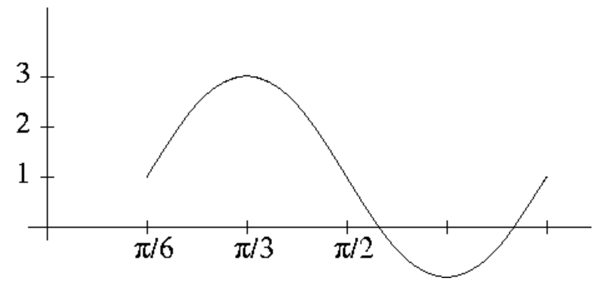
Mathematics 1823-001H Name (please print) ______________________

Examination I

September 26, 2006

Instructions: Give brief, clear answers. It is not expected that most people will be able to answer all the questions, just do what you can in 75 minutes.

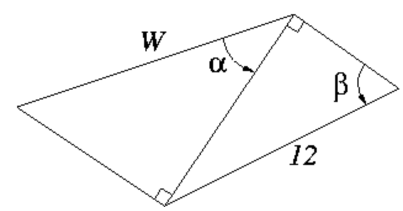
I. (4) The figure to the right shows the graph of a certain function which is obtained from the standard sine function by vertical and horizontal translation and stretching. Determine an expression of the form $y = A\sin(Bx + C) + D$ for this graph.



II. (6) Sketch a graph of the function $\cos(1/x)$. On another coordinate system, sketch a graph of the function $x^2\cos(1/x)$. State the Squeeze Theorem, and explain how it applies to find $\lim_{x\to 0} x^2\cos(1/x)$.

III. (2) Give a precise formal definition of $\lim_{\delta\to L} G(\delta) = x$.

IV. (4) The figure to the right shows two right triangles, with two angles labeled α and β, and a side whose length is shown to be 12. Find an expression involving α and β for the length of the side labeled as W.



V. (4) A rectangular box with volume 7 m^3 has square base and open top. Find the height $h(x)$ of the box and the length $\ell(x)$ of a diagonal of one of its sides as a function of the length x of a side of the base.

VI. (12) Calculate the following limits. Make use of the fact that $\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta} = 1$, when necessary. Give enough explanation to make it clear that you understand where your answer is coming from. Do not use l'Hôpital's Rule.

1. $\lim_{x\to 1}\frac{\sqrt{x} - x^2}{1 - \sqrt{x}}$
2. $\lim_{\theta\to 0}\frac{\sin^2(\theta)}{\theta^2}$
3. $\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta^2}$
4. $\lim_{\theta\to 0}\frac{\sin^2(\theta)}{\theta}$
5. $\lim_{\theta\to 0}\frac{\sin(\theta^2)}{\theta^2}$

VII. (6) For the function $f(x) = x^3$:

1. Write $f(a+h)$ in the form $f(a)+mh+E(h)$ for some expression m involving only a and some function $E(h)$ of h. (Besides just rewriting the expression, tell explicitly what m equals, and what $E(h)$ equals in terms of h.)

2. Find $\lim_{h\to 0} E(h)$, $\lim_{h\to 0} \dfrac{E(h)}{h}$, and $\lim_{h\to 0} \dfrac{E(h)}{h^2}$.

VIII. (3) A certain function $f(x)$ satisfies $\lim_{x\to-\infty} f(x) = 5$. A second function $g(x)$ satisfies $\lim_{x\to 5} g(x) = -\infty$. What is $\lim_{x\to-\infty} (g \circ f)(x)$, and why?

IX. (5) Use the definition of limit to give a rigorous argument that $\lim_{x\to 3} x^2 + x - 4 = 8$. Hint: Use the fact that $x^2 + x - 12 = (x+4)(x-3)$.

X. (2) Give a precise formal definition of $\lim_{x\to\infty} f(x) = -\infty$.

XI. (2) State the Intermediate Value Theorem.

XII. (2) For the function $x^2 + 1$ on the interval $[-5, 0]$ and the intermediate value $N = 11$, find all numbers whose existence is guaranteed by the Intermediate Value Theorem.

XIII. (3) Challenge Problem: Give an example of a function that is defined at every real number, but is continuous only at the point $x = 0$.