Mathematics 1823-001H Name (please print) ____________________

Final Examination

December 13, 2006

I. (5) State the Law of Cosines, and verify it.

II. (7) A lighthouse is located on a small island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute (that is, $\frac{d\theta}{dt} = 8\pi$ radians/min). How fast is the beam of light moving along the shoreline when it is 1 km from P?

III. (4) Find f if $f'''(t) = 60t^2$.

IV. (4) Find f if $f''(x) = 2 + \cos(x)$, $f(0) = -1$, $f(\pi/2) = 0$.

V. (5) Find a function f such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of f at some point.

VI. (7) A farmer wants to fence a rectangular area of 50 hectares and then divide it into thirds with two fences parellel to one of the sides of the field. What are the dimensions of the area that requires the least amount of fence? Remark: 1 hectare is $10,000$ square meters, so a length unit in this problem is 100 meters.

VII. (6) Recall that $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$. For the function $f(x) = x^4$:

1. Write $f(a+h)$ in the form $f(a)+mh+E(h)$ for some expression m involving only a and some function $E(h)$ of h. (Besides just rewriting the expression, tell explicitly what m equals, and what $E(h)$ equals in terms of h.)

2. Find $\lim_{h\to 0} E(h)$, $\lim_{h\to 0} \frac{E(h)}{h}$, and $\lim_{h\to 0} \frac{E(h)}{h^2}$.

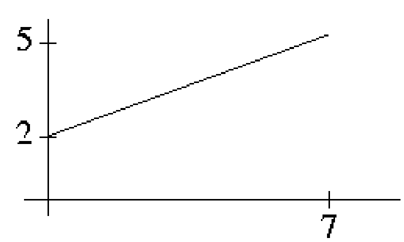
VIII. (10) (a) State the Mean Value Theorem.

(b) Use the Mean Value Theorem to verify that a function with constant zero derivative on an interval must be constant.

(c) Deduce that if two functions on an interval have the same derivative, then one of them is a constant plus the other one.

(d) Show by example that (c) can be false if the domain of the functions is not a connected interval.

IX. (5) To the right is the graph of a linear function f with domain the interval $[0,7]$ and range the interval $[2,5]$.



(a) Find an explicit expression for $A(x)$, the area under the graph of f between 0 and x.

(b) Verify by calculation that $A'(x) = f(x)$.

X. (3) State the Intermediate Value Theorem.

XI. Using calculus, graph the function $2x - \tan(x)$ for $-\pi/2 < x < \pi/2$. Indicate critical points and inflection
(7) points, if any, and any asymptotic behavior.

XII. Determine the following limits (including the possible values ∞ and $-\infty$) by using algebraic manipulation
(9) to put them into a form where the limit is obvious.

(a) $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+2}}$

(b) $\lim_{x\to -\infty} \frac{x}{\sqrt{x^2+2}}$

(c) $\lim_{x\to -\infty} x^{12} - x^{13}$

XIII. Give a precise formal definition of $\lim_{\delta\to L} G(\delta) = x$.
(2)

XIV. Give a precise formal definition of $\lim_{\delta\to -\infty} G(\delta) = \infty$.
(2)

XV. Challenge Problem: Give an explicit example of a continuous function on the interval $(0, 1]$ that has no
(3) minimum value and no maximum value.