Math 1823 homework

16. (10/12) Recall that the rate of change of a function $f(x)$ at the x-value a is the unique number m for which $f(a+h) = f(a)+mh+E(h)$ with $\lim_{h\to 0} \frac{E(h)}{h} = 0$ (if such a number m exists). Use this fact to find the rate of change of the function $\frac{1}{x}$ at a number a as follows.

    1. Fill in the missing details of the following calculation:
    $$\frac{1}{a+h} = \frac{1}{a} + \frac{1}{a+h} - \frac{1}{a} = \frac{1}{a} + \frac{-h}{a^2+ah}$$
    $$= \frac{1}{a} - \frac{h}{a^2} + \frac{-h}{a^2+ah} + \frac{h}{a^2} = \frac{1}{a} - \frac{1}{a^2}h + \frac{ah^2}{a^4+a^3h} \ .$$
    2. Letting $E(h) = \frac{ah^2}{a^4+a^3h}$, check that $\lim_{h\to 0} \frac{E(h)}{h} = 0$.
    3. Deduce that the rate of change of $\frac{1}{x}$ at the x-value a is $-\frac{1}{a^2}$.

17. (10/12) 3.2 # 8, 9, 10, 13

18. (10/12) For these, use the fact that $f'(x) = \lim_{z\to x} \frac{f(z)-f(x)}{z-x}$: 3.2 # 19, 21, 28, 29 [use a different letter from z if you wish]

19. (10/12) For these, use the fact that $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$: 3.2 # 19, 21, 28, 29

20. (10/12) 3.3 as many as needed from # 1-20, 23-32, 35-39

21. (10/12) 3.3 # 33 (use $(1/f)' = -f'/f^2$), 40 (simplify first), 41-42, 53, 58, 62-64, 67-69, 71-74, 87-88

22. (10/12) Use $(1/f)' = -f'/f^2$ or the Quotient Rule, plus the facts that $\frac{d}{dx}(\sin(x)) = \cos(x)$ and $\frac{d}{dx}(\cos(x)) = -\sin(x)$, and any necessary trigonometric identities, to verify that $\tan'(x) = \sec^2(x)$, $\cot'(x) = -\csc^2(x)$, $\sec'(x) = \sec(x)\tan(x)$, and $\csc'(x) = -\csc(x)\cot(x)$.

23. (10/26) as many as needed from 3.5 # 1-16, 21-24.

24. (10/26) 3.5 # 36-44, 46, 47

25. (10/26) as many as needed from 3.6 # 7-46, including at least 3.6 # 25, 26, 31-42

26. (10/26) 3.6 # 55, 56, 63, 64, 71