Math 1823 homework

1. (due 9/5) 1.1 # 12, 15, 19, 47-53

2. (9/5) Draw a diagram (with explanation, of course) illustrating cot(t) and csc(t), analogous to our geometric interpretation of tan(t) and sec(t).

3. (9/5) 1.3 # 6-7, 13-16, 19 (complete the square), 22, 37, 44, 49, 52-54, 57-59, 61(b), 62-64

4. (9/5) Calculate the slopes of the tangent lines to the graph of \( y = x^3 \) as follows.
   (a) Let \( x_0 \) be a fixed positive \( x \)-value, and let \( M(h) \) be the function of \( h \) that is the slope of the line between \( (x_0, x_0^3) \) and \( (x_0 + h, (x_0 + h)^3) \). Calculate \( M(h) \), obtaining the expression
      \[
      M(h) = (3x_0^2 + 3hx_0 + h^2) \frac{h}{h}.
      \]
   (b) Calculate (by completing the square) that \( h^2 + 3x_0h + 3x_0^2 = (h + \frac{3x_0}{2})^2 + \frac{3x_0^2}{4} \)
   (c) For the graph \( y = h^2 \) in the \( y-h \) plane, what is the value of \( y \) when \( h = \frac{3x_0^2}{2} \)?
   (d) Starting from the graph \( y = h^2 \) in the \( y-h \) plane, apply horizontal and vertical translation to the expression in (b) to produce the graph of \( y = M(h) \). The graph will be a parabola, except that the point where the parabola meets the \( y \)-axis is missing. Determine the \( y \)-coordinate of this point.
   (e) Explain as clearly as you can, making use of the graph of \( y = x^3 \) and the graph in (d), why the \( y \)-coordinate found in (d) should be the slope of the tangent line to \( y = x^3 \) at the point \( (x_0, x_0^3) \).

5. (9/5) (Optional. Do this problem only if you think it is fun.) Calculate the slopes of the tangent lines to the graph of \( y = x^3 \) without using a limit argument, as follows.
   (a) Draw a non-tangent, but almost tangent line \( \ell_m \) through the point \( (x_0, x_0^3) \) for a typical positive \( x \)-value \( x_0 \). Using a sketch of the graph of \( y = x^3 \), convince yourself that \( \ell \) crosses \( y = x^3 \) in three points (one of them with \( x \) negative).
   (b) Let \( m \) be the slope of \( \ell_m \). Verify that an equation for \( \ell_m \) is \( y = x_0^3 + m(x - x_0) \).
   (c) The crossing points of \( \ell_m \) and \( y = x^3 \) occur where \( x^3 = x_0^3 + m(x - x_0) \) (why?). Use the algebraic factorization \( x^3 - x_0^3 = (x - x_0)(x^2 + xx_0 + x_0^2) \) to write this equation as \( (x - x_0)(x^2 + xx_0 + (x_0^2 - m)) = 0 \).
   (d) Observe that \( \ell_m \) becomes tangent to \( y = x^3 \) when two of the crossing points merge into one, that is, when the equation in (c) has \( x = x_0 \) as a double root. Since \( x_0 \) already occurs once as the root of the factor \( x - x_0 \), it will be a double root of the equation in (c) exactly when it is a root of \( (x^2 + xx_0 + (x_0^2 - m)) = 0 \). When \( x_0 \) satisfies this equation, what must \( m \) be?