Math 1823 homework

1. (due 9/5) 1.1 # 12, 15, 19, 47-53

2. (9/5) Draw a diagram (with explanation, of course) illustrating $\cot(t)$ and $\csc(t)$, analogous to our geometric interpretation of $\tan(t)$ and $\sec(t)$.

3. (9/5) 1.3 # 6-7, 13-16, 19 (complete the square), 22, 37, 44, 49, 52-54, 57-59, 61(b), 62-64

4. (9/5) Calculate the slopes of the tangent lines to the graph of $y = x^3$ as follows.

   (a) Let x_0 be a fixed positive x-value, and let $M(h)$ be the function of h that is the slope of the line between (x_0, x_0^3) and $(x_0 + h, (x_0 + h)^3)$. Calculate $M(h)$, obtaining the expresssion
   $$M(h) = (3x_0^2 + 3hx_0 + h^2)\frac{h}{h} \ .$$

   (b) Calculate (by completing the square) that $h^2 + 3x_0 h + 3x_0^2 = (h + \frac{3x_0}{2})^2 + \frac{3x_0^2}{4}$

   (c) For the graph $y = h^2$ in the y-h plane, what is the value of y when $h = \frac{3x_0^2}{2}$?

   (d) Starting from the graph $y = h^2$ in the y-h plane, apply horizontal and vertical translation to the expression in (b) to produce the graph of $y = M(h)$. The graph will be a parabola, except that the point where the parabola meets the y-axis is missing. Determine the y-coordinate of this point.

   (e) Explain as clearly as you can, making use of the graph of $y = x^3$ and the graph in (d), why the y-coordinate found in (d) should be the slope of the tangent line to $y = x^3$ at the point (x_0, x_0^3).

5. (9/5) (Optional. Do this problem only if you think it is fun.) Calculate the slopes of the tangent lines to the graph of $y = x^3$ without using a limit argument, as follows.

   (a) Draw a non-tangent, but almost tangent line ℓ_m through the point (x_0, x_0^3) for a typical positive x-value x_0. Using a sketch of the graph of $y = x^3$, convince yourself that ℓ crosses $y = x^3$ in three points (one of them with x negative).

   (b) Let m be the slope of ℓ_m. Verify that an equation for ℓ_m is $y = x_0^3 + m(x - x_0)$.

   (c) The crossing points of ℓ_m and $y = x^3$ occur where $x^3 = x_0^3 + m(x - x_0)$ (why?). Use the algebraic factorization $x^3 - x_0^3 = (x - x_0)(x^2 + xx_0 + x_0^2)$ to write this equation as $(x - x_0)(x^2 + xx_0 + (x_0^2 - m)) = 0$.

   (d) Observe that ℓ_m becomes tangent to $y = x^3$ when two of the crossing points merge into one, that is, when the equation in (c) has $x = x_0$ as a double root. Since x_0 already occurs once as the root of the factor $x - x_0$, it will be a double root of the equation in (c) exactly when it is a root of $(x^2 + xx_0 + (x_0^2 - m)) = 0$. When x_0 satisfies this equation, what must m be?