Math 2513 homework

The remaining homework problems will not be collected, but doing them on an ongoing basis will enable you to derive much more benefit from the lectures, and is your first step in preparing for the final examination.

47. By counting in two different ways (similarly to the the methods used in class to solve problems 4.4 # 22, 29), verify the following identity: for $n \geq 2$, $\binom{n}{2} 2^{n-2} = \sum_{j=2}^{n} \binom{j}{2} \binom{n}{j}$. Hint: count the number of A and B such that $B \subseteq A \subseteq S$ and B has two elements.

48. Obtain the formula from the previous problem by adapting the second method used in class for 4.4 # 29; that is, use the Binomial Theorem to expand $(1+x)^n$, differentiate both sides twice, put $x = 1$, and do a little bit more calculation using the fact that $\binom{j}{2} = \dfrac{j(j-1)}{2}$.

49. 7.1 # 2

50. 7.5 # 2, 3, 10

51. Be able to verify that congruence modulo m is an equivalence relation.

52. 7.5 # 5, 26, 28

53. List explicitly the elements in the congruence classes [0], [1], [2], [3], [4], [5] for the equivalence relation of congruence modulo 6 on the set of integers (show at least five integers from each class). Write out the addition and multiplication tables for the set $\{[0], [1], [2], [3], [4], [5]\}$. Which elements have inverses for multiplication?

54. Let $\sim$ be an equivalence relation on a set A. Prove that for all $x, y \in A$, $x \sim y \Leftrightarrow x \in [y] \Leftrightarrow [x] = [y]$ (use the method of proving $A \Leftrightarrow B \Leftrightarrow C$ by arguing that $A \Rightarrow B$, $B \Rightarrow C$, and $C \Rightarrow A$).

55. 7.5 # 29, 31