Math 2513 homework

37. (11/2) Let S be the set of sequences of 0's and 1's, $S = \{a_1a_2a_3\cdots \mid a_i \in \{0,1\}\}$. A typical element of S is $001011011100011010\cdots$. Adapt the proof that $\mathbb{R}$ is uncountable to prove that S is uncountable.

38. (11/2) 3.3 # 4, 10, 12, 41, 44 (use theorem 10 from section 2.4), 52

39. (11/14) 4.1 # 7-9, 16, 20, 44

40. (11/14) Give counterexamples to the following assertions about the floor and ceiling functions:

    1. $\lceil r \rceil + \lceil s \rceil = \lceil r+s \rceil$
    2. $\lfloor r \rfloor + \lfloor s \rfloor = \lfloor r+s \rfloor$

41. (11/14) Prove the following assertions about the floor and ceiling functions:

    1. $\lceil r \rceil + \lceil s \rceil \geq \lceil r+s \rceil$
    2. $\lfloor r \rfloor + \lfloor s \rfloor \leq \lfloor r+s \rfloor$
    3. $\lceil r \rceil < r+1$
    4. $\lfloor r \rfloor > r-1$

42. (11/14) 4.2 # 2, 3, 8, 9, 20-22

43. (11/14) Study the following argument: "Let $S = \{2,3,4,5,6,7,8,9\}$, and select any five numbers from S. Define $f\colon S \to \{2,3,4,5\}$ by the rule $f(x) = x$ if $2 \leq x \leq 5$ and $f(x) = 11 - x$ if $6 \leq x \leq 9$. By the Pigeonhole Principle, there must be two of the five numbers, call them m and n with $m < n$, for which $f(m) = f(n)$. We cannot have both $m \leq 5$ and $n \leq 5$, since then we would have $m = f(m) = f(n) = n$. Nor can we have $m \geq 6$ and $n \geq 6$, since then we would have $11 - m = f(m) = f(n) = 11 - n$ so again we would have $m = n$. So we must have $m \leq 5$ and $n \geq 6$. But then, $m = f(m) = f(n) = 11 - n$, so $m + n = 11$." Now, adapt the argument to verify that the answers to problems 4.2 # 15-16 are 4 and 5.

44. (11/14) 4.3 # 3, 5, 9, 13

The remaining homework problems will not be collected, but doing them on an ongoing basis will enable you to derive much more benefit from the lectures, and is your first step in preparing for the final examination.

45. 4.3 # 15-18, 26, 27

46. 4.4 # 12, 21