Math 2513 homework

17. (10/10) Take as known the fact that \( \sin: \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \to [-1, 1] \) is bijective.

1. Sketch the graph of this function in the \( x-y \) plane.
2. Let \( \sin^{-1} \) be the inverse of this function. Tell the domain and codomain of \( \sin^{-1} \), and sketch the graph of \( \sin^{-1} \) in the \( x-y \) plane.
3. For which \( x \) is \( \sin(\sin^{-1}(x)) = x? \)
4. For which \( x \) is \( \sin^{-1}(\sin(x)) = x? \)
5. Draw a right triangle whose sides are 1, \( x \), and \( \sqrt{1-x^2} \), and label which angle is \( \sin^{-1}(x) \). Use the triangle to find an expression for \( \cos(\sin^{-1}(x)) \).
6. Use the identity \( \sin(\sin^{-1}(x)) = x \), the chain rule, and the expression for \( \cos(\sin^{-1}(x)) \) to obtain the formula \( \frac{d}{dx} \left( \sin^{-1}(x) \right) = \frac{1}{\sqrt{1-x^2}} \). Write this formula as an integration formula.

18. (10/17) 2.4 Prove part 3 of Theorem 1, \# 6, 7, 12, 13

19. (10/17) Be able to write out Euler’s proof that there are infinitely many primes from memory.

20. (10/17) 2.4 \# 16, 17, 28, 29

21. (10/24) 2.4 \# 30, 32, 38-45

22. (10/24) 2.4 \# 46

23. (10/24) For each integer \( n \) with \( 0 \leq n < 12 \), use trial and error to find an integer \( m \) (with \( 0 \leq m < 12 \)) for which \( 5m \equiv n \mod 12 \).

24. (10/24) Use the fact that \( 5 \cdot 5 \equiv 1 \mod 12 \) to solve the previous problem in a much better way.

25. (10/24) This problem will help you understand the theorem that if \( a \equiv b \mod m \) and \( c \equiv d \mod m \), then \( a + c \equiv b + d \mod m \). Define a junk relation \( a \text{R} b \) by \( a \text{R} b \iff (a - b)^2 \leq 25 \).

1. Prove that for all integers \( a \), \( a \text{R} a \).
2. Prove that for all integers \( a \) and \( b \), if \( a \text{R} b \) then \( b \text{R} a \).
3. Find counterexample to the following assertion:
   If \( a \text{R} b \) and \( c \text{R} d \), then \( a + c \equiv b + d \mod m \).
4. Prove that if \( a + c \equiv b + d \mod m \), then \( a \text{R} b \).
5. Let \( \sim \) be a relation on integers that satisfies \( a \sim a \) for all \( a \). Prove that if \( a \sim b \) and \( c \sim d \) imply that \( a + c \sim b + d \), then \( a + c \sim b + c \) implies that \( a \sim b \).
6. Disprove the converse of the previous statement.