Math 2513 homework

17. (10/10) Take as known the fact that $\sin\colon [-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1, 1]$ is bijective.

    1. Sketch the graph of this function in the x-y plane.
    2. Let $\sin^{-1}$ be the inverse of this function. Tell the domain and codomain of $\sin^{-1}$, and sketch the graph of $\sin^{-1}$ in the x-y plane.
    3. For which x is $\sin(\sin^{-1}(x)) = x$?
    4. For which x is $\sin^{-1}(\sin(x)) = x$?
    5. Draw a right triangle whose sides are 1, x, and $\sqrt{1-x^2}$, and label which angle is $\sin^{-1}(x)$. Use the triangle to find an expression for $\cos(\sin^{-1}(x))$.
    6. Use the identity $\sin(\sin^{-1}(x)) = x$, the chain rule, and the expression for $\cos(\sin^{-1}(x))$ to obtain the formula $\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$. Write this formula as an integration formula.

18. (10/17) 2.4 Prove part 3 of Theorem 1, # 6, 7, 12, 13

19. (10/17) Be able to write out Euler's proof that there are infinitely many primes from memory.

20. (10/17) 2.4 # 16, 17, 28, 29

21. (10/24) 2.4 # 30, 32, 38-45

22. (10/24) 2.4 # 46

23. (10/24) For each integer n with $0 \le n < 12$, use trial and error to find an integer m (with $0 \le m < 12$) for which $5m \equiv n \mod 12$.

24. (10/24) Use the fact that $5 \cdot 5 \equiv 1 \mod 12$ to solve the previous problem in a much better way.

25. (10/24) This problem will help you understand the theorem that if $a \equiv b \mod m$ and $c \equiv d \mod m$, then $a + c \equiv b + d \mod m$. Define a junk relation $a \mathrel{\mathrm{R}} b$ by $a \mathrel{\mathrm{R}} b \Leftrightarrow (a-b)^2 \le 25$.

    1. Prove that for all integers a, $a \mathrel{\mathrm{R}} a$.
    2. Prove that for all integers a and b, if $a \mathrel{\mathrm{R}} b$ then $b \mathrel{\mathrm{R}} a$.
    3. Find counterexample to the following assertion:
       If $a \mathrel{\mathrm{R}} b$ and $c \mathrel{\mathrm{R}} d$, then $a + c \mathrel{\mathrm{R}} b + d$.
    4. Prove that if $a + c \mathrel{\mathrm{R}} b + c$, then $a \mathrel{\mathrm{R}} b$.
    5. Let $\sim$ be a relation on integers that satisfies $a \sim a$ for all a. Prove that if $a \sim b$ and $c \sim d$ imply that $a + c \sim b + d$, then $a + c \sim b + c$ implies that $a \sim b$.
    6. Disprove the converse of the previous statement.