Mathematics 2433-001H — Name (please print) ____________________

Examination III

November 18, 2005

I. (5) Find the Taylor series for the function $f(x) = x^3$ at $a = -1$.

II. (15) Use power series to calculate each of the following:

1. $\displaystyle\lim_{x\to 0} \frac{\ln(1+3x) - 3x}{x^2}$.

2. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$

3. A numerical series whose value is $\displaystyle\int_0^1 \frac{\sin(x)}{x}\,dx$ (but do not try to calculate the numerical value of the series).

III. (6) For a certain function $f(x)$, the Taylor polynomial T_5 of degree 5 at $a = 0$ is $1 + 20x^3 + x^5$. Suppose that one uses the value $T_5(0.1) = 1.02001$ as an approximation for $f(0.1)$. Suppose that it is known that all derivatives of f have values between 0 and 200 at x-values in the range $-10 \le x \le 10$. Use Taylor's form $R_n(x) = \displaystyle\int_a^x \frac{(x-t)^n}{n!} f^{(n+1)}(t)\,dt$ for the remainder to calculate an upper bound for the error of this approximation (that is, make an upper estimate of $|R_5(0.1)|$).

IV. (6) We know that $\vec{\jmath} \times \vec{\imath} = -\vec{k}$ and $\vec{k} \times \vec{\imath} = \vec{\jmath}$.

1. Use these facts to find a vector $\vec{v}$ such that $\vec{v} \times \vec{\imath} = \vec{\jmath} + 2\,\vec{k}$.

2. Find a vector $\vec{w}$ such that $\vec{w} \times (\vec{w} + 2\vec{\imath})$ equals $\vec{\jmath} + 2\,\vec{k}$.

V. (12) A straight line L has direction vector $2\vec{\imath} - \vec{\jmath} + \vec{k}$ and passes through the point $P = (0, 4, -0.5)$.

1. Write an equation for L as a vector-valued function of t.

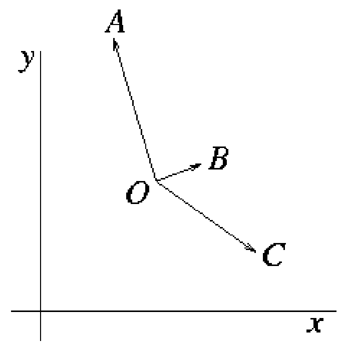
2. Write parametric equations for L.

3. Write equations in symmetric form for L.

4. Write an equation for the plane through P perpendicular to L.

5. Find a normal vector to the plane that contains L and also contains the origin.

VI. (12) Four points O, A, B, and C in the xy-plane are shown in the figure to the right; the angle AOB is a right angle. Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be the vectors from O to A, B, and C respectively.



1. Tell why $\vec{a} \cdot \vec{b} > \vec{a} \cdot \vec{c}$.

2. Tell why $\|\,\vec{a} \times \vec{b}\,\| < \|\,\vec{a} \times \vec{c}\,\|$.

3. Tell why $(\vec{c} \times \vec{a}) \times \vec{b}$ points in the direction of $\vec{a}$.

4. Tell the approximate direction of $\vec{c} \times (\vec{a} \times \vec{b})$.