Mathematics 2433-001H Name (please print) ____________

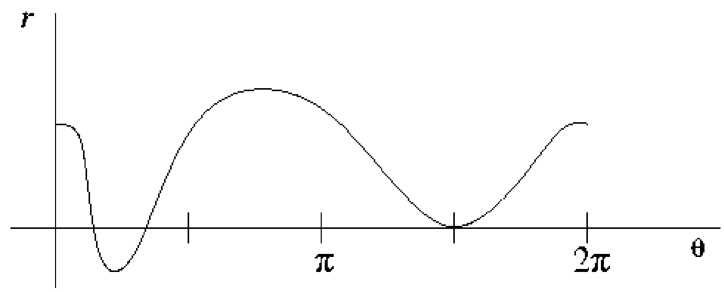
Examination I

September 21, 2005

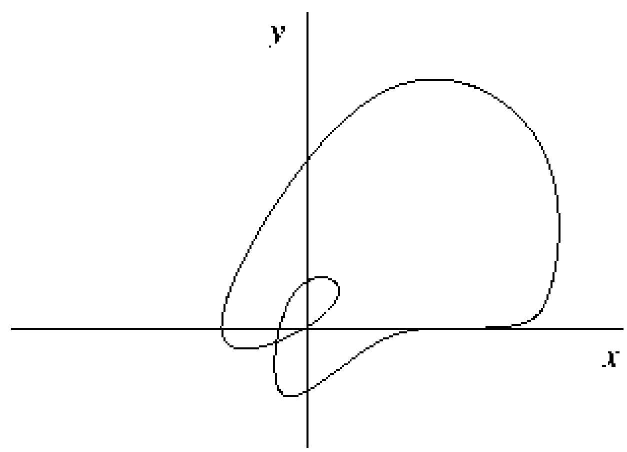
I. (6) The sequence whose n^{th} term is $\dfrac{(-1)^n n^2}{1+n^3}$ converges to 0. State the Squeeze Principle for limits of sequences, and use it to verify that this sequence converges to 0.

II. (5) Sketch the following curve and indicate with an arrow the direction in which the curve is traced as the parameter moves: $x = \cosh(t)$, $y = \sinh(t)$. Give a brief explanation of why this is the graph.

III. (5) The graph of a certain equation $r = f(\theta)$ is shown at the right, in a rectangular θ-r coordinate system. In an x-y coordinate system, make a reasonably accurate graph of the polar equation $r = f(\theta)$ for this function.



IV. (5) The graph of a certain polar equation $r = f(\theta)$ is shown at the right, in an rectangular x-y coordinate system. In a rectangular θ-r coordinate system, make a reasonably accurate graph of the rectangular equation $r = f(\theta)$ for this function. Assume that $r = 1$ when $\theta = 0$.



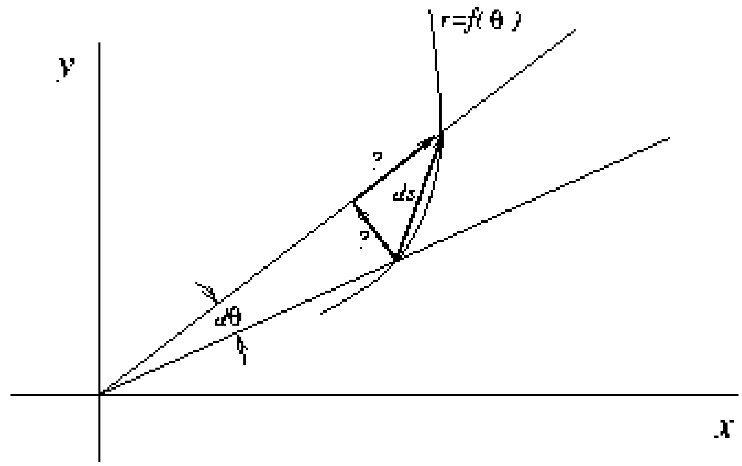
V. (6) Give (without extensive verification) examples of the following:

(a) A sequence a_n such that $\{|a_n|\}$ is monotonic, but $\{a_n\}$ is not monotonic.

(b) A sequence a_n such that $\{a_n\}$ is monotonic, but $\{|a_n|\}$ is not monotonic.

VI. (4) A certain decreasing sequence $\{a_n\}$ has all $a_n > 0$. Explain how one knows that it must converge.

VII. The figure to the right shows the graph of a polar equation $r = f(\theta)$ in the x-y plane, and an arrow representing the differential of arclength ds, express the two differentials indicated by ? in terms of the differential $d\theta$, and then use them and the Pythagorean theorem to calculate ds in terms of $d\theta$.
(7)



VIII. The line $y = x$ is parameterized by $x = t^3/3$, $y = t^3/3$ for t in the domain of all real numbers.
(8)

(a) Calculate ds and use it to find the distance traveled between times $t = -1$ and $t = 1$.

(b) The chain rule $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ gives the expression $\frac{dy}{dx} = \frac{t^2}{t^2}$. This expression is undefined when $t = 0$. Using the interpretation of the parametric equations as describing the motion of a point P that moves with coordinates $(x, y) = (t^3/3, t^3/3)$, explain why it is reasonable for $\frac{dy}{dx}$ to be undefined when $t = 0$.

IX. A point P moves according to the *polar* parametric equations $\theta = \sin(t)$, $r = t$. Describe the motion for $0 \le t \le 314.159$. A sketch will certainly be helpful, as will the fact that 1 radian is approximately 57 degrees.
(5)