Math 5853 homework

Instructions: All problems should be prepared for presentation at the problem sessions. If a problem has a due date listed, then it should be written up formally and turned in on the due date.

9. (due 9/7) Prove that \( \mathbb{R} \) with the lower limit topology is Hausdorff.

10. Exercises 1.9.1-1.9.6

11. (due 9/7) Let \( B_1 \) and \( B_2 \) be bases for topologies \( T_1 \) and \( T_2 \) on \( X \). Suppose that for every \( B_1 \in B_1 \) and every \( x \in B_1 \), there exists \( B_2 \in B_2 \) such that \( x \in B_2 \subseteq B_1 \). Prove that \( T_1 \subseteq T_2 \).

12. Exercises 1.9.7, 1.9.8

13. (9/14) Prove a refined version of the Basis Recognition Theorem: Let \( X \) be a topological space and let \( B \) be a basis for the topology on \( X \). Let \( C \) a collection of subsets of \( X \). Then \( C \) is a basis for the topology on \( X \) if and only if

   1. for each \( C \in C \), \( C \) is open in \( X \), and
   2. for each element \( B \in B \) and each \( x \in B \), there exists \( C \in C \) such that \( x \in C \subseteq B \).

14. (9/14) Suppose that \( X \) is a topological space and \( B \) is a basis for the topology on \( X \). If \( A \subseteq X \), then \( \{ B \cap A \mid B \in B \} \) is a basis for the subspace topology on \( A \).

15. Let \( (X, d) \) be a metric space (with the metric topology) and let \( A \subseteq X \). Show that the subspace topology on \( A \) equals the metric topology on \( A \) for the metric \( d|_{A \times A} \).

16. (9/14) Exercise 1.2.8.

17. Exercises 1.2.9, 1.2.10.

18. (9/14) Exercises 1.9.27, 1.9.29, 1.9.30.

19. Exercises 1.9.15-1.9.16.

20. Exercises 1.9.11-1.9.13, 1.9.17.