Math 5853 homework

Instructions: All problems should be prepared for presentation at the problem sessions. If a problem has a due date listed, then it should be written up formally and turned in on the due date.

9. (due 9/7) Prove that $\mathbb{R}$ with the lower limit topology is Hausdorff.

10. Exercises 1.9.1-1.9.6

11. (due 9/7) Let $\mathcal{B}_1$ and $\mathcal{B}_2$ be bases for topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on X. Suppose that for every $B_1 \in \mathcal{B}_1$ and every $x \in B_1$, there exists $B_2 \in \mathcal{B}_2$ such that $x \in B_2 \subseteq B_1$. Prove that $\mathcal{T}_1 \subseteq \mathcal{T}_2$.

12. Exercises 1.9.7, 1.9.8

13. (9/14) Prove a refined version of the Basis Recognition Theorem: Let X be a topological space and let $\mathcal{B}$ be a basis for the topology on X. Let $\mathcal{C}$ a collection of subsets of X. Then $\mathcal{C}$ is a basis for the topology on X if and only if

    1. for each $C \in \mathcal{C}$, C is open in X, and
    2. for each element $B \in \mathcal{B}$ and each $x \in B$, there exists $C \in \mathcal{C}$ such that $x \in C \subseteq B$.

14. (9/14) Suppose that X is a topological space and $\mathcal{B}$ is a basis for the topology on X. If $A \subset X$, then $\{B \cap A \mid B \in \mathcal{B}\}$ is a basis for the subspace topology on A.

15. Let (X, d) be a metric space (with the metric topology) and let $A \subseteq X$. Show that the subspace topology on A equals the metric topology on A for the metric $d|_{A\times A}$.

16. (9/14) Exercise 1.2.8.

17. Exercises 1.2.9, 1.2.10.

18. (9/14) Exercises 1.9.27, 1.9.29, 1.9.30.

19. Exercises 1.9.15-1.9.16.

20. Exercises 1.9.11-1.9.13, 1.9.17.

21. (9/21) Exercises 1.9.22, 1.9.23.