Mathematics 1823-001H | Name (please print) ______________________

Examination III

December 2, 2004

Instructions: Give brief, clear answers. It is not expected that you will be able to do all the problems. Just relax and do your best.

I. (5) State the Mean Value Theorem, including its standard hypotheses.

II. (5) For the function $f(x) = \sqrt{x} - x$, find the value of c that satisfies the Mean Value Theorem when $a = 0$ and $b = 3$.

III. (15) Using the Mean Value Theorem, verify each of the following assertions, assuming that f and g are functions that are differentiable for all x, and that a and b are numbers with $a < b$:

1. If $f'(x) \leq 0$ for all x, then $f(a) \geq f(b)$.
2. If $f'(x) = g'(x)$ for all x, then there is a number C so that $g(x) = f(x) + C$ for all x.
3. If $f''(x) > 0$ for all x, then for $a < x < b$ the graph of $f(x)$ lies above the tangent line to $y = f(x)$ at the point $(a, f(a))$.

IV. (6) State the Extreme Value Theorem. Give an example of a function $f(x)$ defined on $[0, 1]$ that has no maximum value.

V. (6) Define a function $f(x)$ by $f(x) = x^2$ if $x \leq 1$, $f(x) = 1$ if $1 \leq x \leq 2$, and $f(x) = 3 - x$ if $x \geq 2$.

1. Tell all x-values, if any, where f has a local minimum.
2. Tell all x-values, if any, where f has a local maximum.

VI. (7) Analyze the function $f(x) = \sqrt{\dfrac{x-5}{x}}$, determining its noteworthy features and where they occur, and use this information to sketch the graph of $f(x)$.

VII. (7) Analyze the function $f(x) = \sin(x) - \tan(x)$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$, determining its noteworthy features and where they occur, and use this information to sketch the graph of $f(x)$ for these x-values.

VIII. (8) A box with open top is constructed from a square piece of cardboard, 3 meters on a side, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

IX. (5) A point travels from the origin $(0, 0)$ to the point $(1, 0)$, by first traveling in a straight line from $(0, 0)$ to a point $(x, 1)$ at a speed of 3 (units of distance per second), then traveling in a straight line from $(x, 1)$ to $(1, 0)$ at a speed of 2. Write an expression for the total required time $T(x)$, but do not try to find its minimum value.

X. (5) Suppose that a function $f(x)$ has a horizontal asymptote $y = L$ as $x \to \infty$. Is it necessarily true that $\lim_{x\to\infty} f'(x) = 0$? Either explain why it is true (if you think it is true), or show how it could be false (if you think it is false).

XI. (6) Recall that $f'(a)$ is the unique number, if such a number exists, for which $f(a+h) = f(a) + \Big(f'(a) + \dfrac{\epsilon(h)}{h}\Big)h$ with $\lim_{h\to 0} \dfrac{\epsilon(h)}{h} = 0$. Assuming that $f'(a)$ exists and $f'(a) > 0$, verify that there exists $\delta > 0$ so that if $a < x < a + \delta$ then $f(a) < f(x)$.