Math 1823 homework

Instructions: Work the assigned problems. Book problems shown in **boldface** should be written up formally and turned in no later than the due date.

7. (due 9/21) 2.5 # 1, **43, 44, 59, 61**

8. (9/21) 2.6 # **8, 10, 16**

9. (9/21) Determine the rate of change of the cosine function by drawing a careful diagram of the points $(\cos(a), \sin(a))$ and $(\cos(x), \sin(x))$ and nearby distances and angles, then using the diagram to argue that for x near a, $\cos(x) - \cos(a)$ is very near $-\sin(a)\,(x-a)$, and then obtaining $\lim_{x\to a} \dfrac{\cos(x)-\cos(a)}{x-a}$ from this observation.

10. (9/21) Think of the function $f(x) = x^3$ as being the volume of a cube of side x. Draw a cube of side a, and explain (with pictures, of course) the volume that is added on when the side is increased to $a+h$ (there are seven parts added on, three of volume a^2h, three of volumes ah^2, and one of volume h^3). Use this viewpoint to show that the rate of change of f at a is $3a^2$. Give an exact expression for the error of linear approximation at a, that is, the function we call $\epsilon(h)$.

11. (9/21) Recall that the rate of change of a function $f(x)$ at the x-value a is the unique number m for which $f(a+h) = f(a) + mh + \epsilon(h)$ with $\lim_{h\to 0} \dfrac{\epsilon(h)}{h} = 0$ (if such a number m exists). Use this fact to find the rate of change of the function $\dfrac{1}{x}$ at a number a as follows.

    1. Fill in the missing details of the following calculation:
    $$\frac{1}{a+h} = \frac{1}{a} + \frac{1}{a+h} - \frac{1}{a} = \frac{1}{a} + \frac{-h}{a^2+ah}$$
    $$= \frac{1}{a} - \frac{h}{a^2} + \frac{-h}{a^2+ah} + \frac{h}{a^2} = \frac{1}{a} - \frac{1}{a^2}h + \frac{ah^2}{a^4+a^3h} \ .$$

    2. Letting $\epsilon(h) = \dfrac{ah^2}{a^4+a^3h}$, check that $\lim_{h\to 0}\dfrac{\epsilon(h)}{h} = 0$.

    3. Deduce that the rate of change of $\dfrac{1}{x}$ at the x-value a is $-\dfrac{1}{a^2}$.