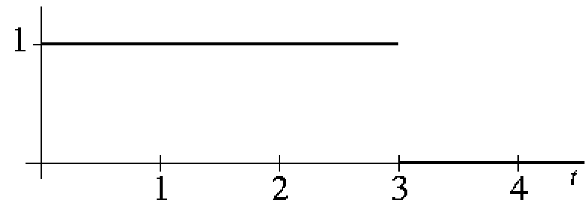
Mathematics 3113-001 Name (please print) ______________________

Examination III Form A

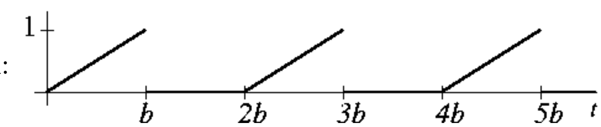
November 20, 2003

I. The figure to the right shows the graph of a function $f(t)$. Using the *definition* of the Laplace transform to calculate $\mathcal{L}\{f(t)\}$.
(4)



II. For the function $f(t)$ shown in the previous problem, write an expression for $f(t)$ using the Heaviside step function $u_3(t)$, and use this expression and formulas from your table of Laplace transforms to calculate the Laplace transform of $f(t)$.
(4)

III. Calculate the Laplace transform of this periodic function:
(6)



IV. Calculate the inverse Laplace transforms of the following functions.
(16)

1. $\dfrac{1}{(s-5)^4}$

2. $\dfrac{s^2}{(s-1)(s-2)(s-3)}$

3. $\dfrac{s(1-e^{-3s})}{s^2+\pi^2}$ (graph the resulting $f(t)$)

4. $\ln\left(1+\dfrac{1}{s^2}\right)$ (hint: it is best to simplify first, using properties of the logarithm function)

V. (8) Use Laplace transforms to solve the following system of differential equations. Carry out the inverse transform without using partial fractions.
$x' = x + y,\ y' = 3x - y,\ \ x(0) = 1,\ y(0) = 0$

VI. (6) Use Laplace transforms to solve the following initial value problem, and **graph the solution:**
$x^{(3)} = 2\,\delta(t-1),\ \ x(0) = x'(0) = x''(0) = 0$

VII. Use Laplace transforms to solve the following initial value problem. Express the answer using an integral
(6) involving $f(t)$: $x'' + x = f(t),\ \ x(0) = x'(0) = 0$.

VIII. Use the definition of convolution to calculate $(t+1) * t$.
(4)

IX. Write a function $f(t)$ whose derivative is e^{-t^2} and for which $f(0) = \sqrt{13}$.
(3)