Mathematics 3113-001 Name (please print) ______________________

Examination II Form B

October 23, 2003

I. (5) Write $2\cos(3t) + \sin(3t)$ in phase-angle form, leaving an expression involving the inverse tangent function $\tan^{-1}$ in the answer (i. e. do not perform a numerical approximation).

II. (10) Use the method of variation of parameters to find a particular solution of the differential equation $y'' + y' = x$ as follows.

1. Given that $y_1 = 1$ and $y_2 = e^{-x}$ are two linearly independent solutions of the associated homogeneous equation $y'' + y' = 0$, write the general equations $y_1 u_1' + y_2 u_2' = 0$, $y_1' u_1' + y_2' u_2' = f(x)$ of the method of variation of parameters to find u_1' and u_2' for the nonhomogeneous linear equation $y'' + y' = x$.

2. Solve for u_1' and find u_1.

3. Solve for u_2'. Use the integration formula $\int xe^x \, dx = (x-1)e^x$ to find u_2.

4. Write the particular solution y_p that has been found.

III. (8) Using the formula $x^s\big((A_0 + A_1x + \cdots + A_mx^m)e^{rx}\cos(kx) + (B_0 + B_1x + \cdots + B_mx^m)e^{rx}\sin(kx)\big)$, write trial solutions for the method of undetermined coefficients for the following differential equations, but *do not* substitute them into the equations or proceed further with finding the solution.

1. $y'' + y' = e^x \sin(2x)$

2. $y'' + y' = x^2e^{-x}$

IV. (5) Transform the differential equation $t^3x^{(3)} - 2t^2x'' + 3tx' + 5x = \ln(t)$ into an equivalent system of first-order differential equations.

V. (8) Given that $r^{12} + 3r^{11} + 6r^{10} + 9r^9 + 10r^8 + 9r^7 + 6r^6 + 3r^5 + r^4 = r^4(r+1)(r^2+r+1)(r^2+1)^2(r+1)$, write the general solution to $y^{(12)} + 3y^{(11)} + 6y^{(10)} + 9y^{(9)} + 10y^{(8)} + 9y^{(7)} + 6y^{(6)} + 3y^{(5)} + y^{(4)} = 0$.

VI. Consider the boundary-value problem $y'' + \lambda y = 0, \;\; y(a) = a_0, \; y(b) = b_0$.
(4)

1. Define what it means to say that a value of λ *is an eigenvalue* of this problem.

2. Define what it means to say that a function y *is an eigenfunction* associated to an eigenvalue λ.

VII. Use the method of elimination to solve the linear system. Use differential operators, if you prefer, or else
(6) just solve the second equation for x and use that expression in the first equation.

$$x' = x - 2y,$$
$$y' = 2x - 3y$$

VIII. Consider the boundary-value problem $y'' + \lambda y = 0,\ \ y(0) = 0,\ y(\pi) = 0.$
(10)

1. Show that $\lambda = 0$ is not an eigenvalue of this problem.

2. By writing $\lambda = \alpha^2$, $\alpha > 0$, find all positive eigenvalues, and an associated eigenfunction for each positive eigenvalue.

IX. (Bonus problem) Simplify $(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}})^{100}$.
(4)