Mathematics 3113-001 Name (please print) ______________________

Examination II Form A

October 23, 2003

I. (8) Using the formula $x^s\big((A_0 + A_1x + \cdots + A_mx^m)e^{rx}\cos(kx) + (B_0 + B_1x + \cdots + B_mx^m)e^{rx}\sin(kx)\big)$, write trial solutions for the method of undetermined coefficients for the following differential equations, but *do not* substitute them into the equations or proceed further with finding the solution.

1. $y'' + y' = e^x \cos(3x)$

2. $y'' + y' = x^2e^{-x}$

II. (8) Given that $r^{12} + 3r^{11} + 6r^{10} + 9r^9 + 10r^8 + 9r^7 + 6r^6 + 3r^5 + r^4 = r^4(r+1)(r^2+r+1)(r^2+1)^2(r+1)$, write the general solution to $y^{(12)} + 3y^{(11)} + 6y^{(10)} + 9y^{(9)} + 10y^{(8)} + 9y^{(7)} + 6y^{(6)} + 3y^{(5)} + y^{(4)} = 0$.

III. (5) Transform the differential equation $t^3x^{(3)} - 2t^2x'' + 3tx' + 5x = \ln(t)$ into an equivalent system of first-order differential equations.

IV. (10) Use the method of variation of parameters to find a particular solution of the differential equation $y''+y' = x$ as follows.

1. Given that $y_1 = 1$ and $y_2 = e^{-x}$ are two linearly independent solutions of the associated homogeneous equation $y'' + y' = 0$, write the general equations $y_1u_1' + y_2u_2' = 0$, $y_1'u_1' + y_2'u_2' = f(x)$ of the method of variation of parameters to find u_1' and u_2' for the nonhomogeneous linear equation $y'' + y' = x$.

2. Solve for u_1' and find u_1.

3. Solve for u_2'. Use the integration formula $\int xe^x\,dx = (x-1)e^x$ to find u_2.

4. Write the particular solution y_p that has been found.

V. (5) Write $3\cos(3t) + \sin(3t)$ in phase-angle form, leaving an expression involving the inverse tangent function $\tan^{-1}$ in the answer (i. e. do not perform a numerical approximation).

VI. (6) Use the method of elimination to solve the linear system. Use differential operators, if you prefer, or else just solve the second equation for x and use that expression in the first equation.

$$x' = x - 2y,$$
$$y' = 2x - 3y$$

VII. (4) Consider the boundary-value problem $y'' + \lambda\, y = 0, \;\; y(a) = a_0, \; y(b) = b_0$.

1. Define what it means to say that a value of λ *is an eigenvalue* of this problem.

2. Define what it means to say that a function y *is an eigenfunction* associated to an eigenvalue λ.

VIII. Consider the boundary-value problem $y'' + \lambda y = 0,\;\; y(0) = 0,\; y(\pi) = 0.$
(10)

1. Show that $\lambda = 0$ is not an eigenvalue of this problem.

2. By writing $\lambda = \alpha^2$, $\alpha > 0$, find all positive eigenvalues, and an associated eigenfunction for each positive eigenvalue.

IX. (Bonus problem) Simplify $(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}})^{100}$.
(4)