Formulas for the Laplace Transform

$$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}(t^\alpha) = \frac{\Gamma(\alpha + 1)}{s^{\alpha+1}}, \text{ where } \Gamma(x) = \int_0^\infty e^{-t} t^{x-1} \, dt$$

$$\mathcal{L}(e^{at}) = \frac{1}{s - a}$$

$$\mathcal{L}(\cos(at)) = \frac{s}{s^2 + a^2}$$

$$\mathcal{L}(\sin(at)) = \frac{a}{s^2 + a^2}$$

$$\mathcal{L}(\cosh(at)) = \frac{s}{s^2 - a^2}$$

$$\mathcal{L}(\sinh(at)) = \frac{a}{s^2 - a^2}$$

$$\mathcal{L}\left(\frac{1}{2a} t \sin(at)\right) = \frac{s}{(s^2 + a^2)^2}$$

$$\mathcal{L}\left(\frac{1}{2a^3} (\sin(at) - at\cos(at))\right) = \frac{1}{(s^2 + a^2)^2}$$

$$\mathcal{L}(u_a(t)) = \frac{e^{-as}}{s}, \text{ where } u_a(r) = 0 \text{ for } r < a \text{ and } u_a(r) = 1 \text{ for } r > a$$

$$\mathcal{L}(f^{(n)}(t)) = s^n \mathcal{L}(f(t)) - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) - \cdots - s f^{(n-2)}(0) - f^{(n-1)}(0)$$

$$\mathcal{L}\left(\int_0^t f(\tau) \, d\tau\right) = \frac{1}{s} \mathcal{L}(f(t))$$

$$\mathcal{L}(e^{at} f(t)) = F(s - a)$$

$$\mathcal{L}(t^n f(t)) = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$\mathcal{L}\left(\frac{f(t)}{t}\right) = \int_s^\infty F(\sigma) \, d\sigma$$

$$\mathcal{L}((f * g)(t)) = F(s) \, G(s), \text{ where } (f * g)(t) = \int_0^t f(\sigma) g(t - \sigma) \, d\sigma$$

$$\mathcal{L}(f(t)) = \frac{1}{1 - e^{-ps}} \int_0^p e^{-st} f(t) \, dt, \text{ if } f(t + p) = f(t) \text{ for all } t$$

$$\mathcal{L}(u_a(t) f(t - a)) = e^{-as} F(s)$$

$$\mathcal{L}(u(t - a) f(t - a)) = e^{-as} F(s), \text{ where } u(r) = 0 \text{ for } r < 0 \text{ and } u(r) = 1 \text{ for } r > 0$$

$$\mathcal{L}(\delta_a(t)) = e^{-as}, \text{ where } \delta_a(t) \text{ is the Dirac } \delta \text{ function (this is often written using } \delta(t - a) \text{ to mean } \delta_a(t))$$