Mathematics 2443-005 Name (please print) ____________________

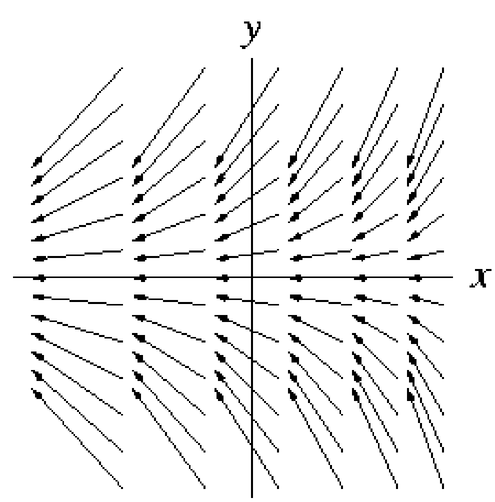
Examination III

November 19, 2002

I. (5) Use Green's theorem to calculate $\int_C (e^y\,\vec{\imath} + xy\,\vec{\jmath}) \cdot d\vec{r}$, where C is the boundary of the unit square $0 \le x \le 1$, $0 \le y \le 1$, with the *clockwise* orientation.

II. (5) The vector field $\vec{F}(x,y,z) = (1+2xz)\,\vec{\imath} + (1+\cos(z))\,\vec{\jmath} + (x^2 - y\sin(z))\,\vec{k}$ is known to be conservative. Find a function $f(x,y,z)$ so that $\nabla f = \vec{F}$.

III. (6) The figure to the right shows a vector field $\vec{F}(x,y) = P(x,y)\vec{\imath} + Q(x,y)\vec{\jmath}$ in the xy-plane.



1. Determine whether each of $\dfrac{\partial P}{\partial x}$, $\dfrac{\partial P}{\partial y}$, $\dfrac{\partial Q}{\partial x}$, $\dfrac{\partial Q}{\partial y}$ is positive, negative, or 0.

2. Use Green's Theorem to verify that $\int_C \vec{F} \cdot d\vec{r} = 0$ for any simple closed curve C.

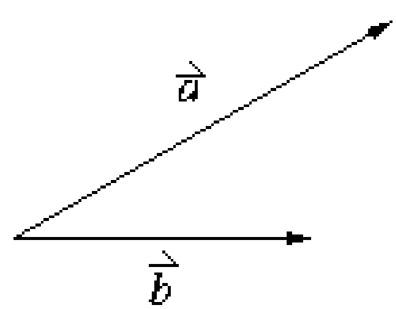
IV. Calculate each of the following.
(9)

1. $\text{curl}(xz\,\vec{\imath}+xz\,\vec{\jmath}+xy\,\vec{k})$

2. $\text{div}(\nabla f)$, where f is a function of (x, y, z)

3. $\int_C \nabla f \cdot d\vec{r}$, where $f(x, y) = \sin(\sin(xy))$ and C is the line segment in the xy-plane from $(0, 0)$ to $(1, \pi/3)$.

V. The figure to the right shows two vectors, $\vec{a}$ and $\vec{b}$.
(6)



1. Give a general formula for the unit vector in the direction of $\vec{a}$.

2. Draw the vector projection of $\vec{b}$ to $\vec{a}$.

3. Give a general formula for the scalar projection of $\vec{b}$ to $\vec{a}$.

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VI. Let T be the sphere of radius a centered at the origin. Recall that T can be parameterized by $x = a\cos(\theta)\sin(\phi)$, $y = a\sin(\theta)\sin(\phi)$, and $z = a\cos(\phi)$.
(9)

1. What is the domain R for this parameterization? Draw it in the (θ, ϕ)-plane.

2. Calculate $\vec{r}_\phi$ and $\vec{r}_\theta$ for this parameterization.

3. Given that $\vec{r}_\phi \times \vec{r}_\theta = a\ \sin(\phi)(x\vec{\imath} + y\vec{\jmath} + z\vec{k})$, calculate $\| \vec{r}_\phi \times \vec{r}_\theta \|$.

4. Write a double integral of a function of (θ, ϕ) on R whose value equals $\iint_T x^2\, dS$, but do not try to calculate the value.

VII. Let S be the surface $z = f(x, y)$, where the domain of $f(x, y)$ is the unit square $0 \leq x \leq 1$ and $0 \leq y \leq 1$.
(8) Parameterize S by $x = u$, $y = v$, and $z = f(u, v)$, where the domain of the parameterization is the unit square R given by $0 \leq u \leq 1$ and $0 \leq v \leq 1$ in the uv-plane.

1. Calculate $\vec{r}_u$ and $\vec{r}_v$.

2. Calculate $\vec{r}_u \times \vec{r}_v$.

3. Calculate $\| \vec{r}_u \times \vec{r}_v \|$.

4. Express dS in terms of dR.

VIII. Let C consist of the line segment from $(0, 0)$ to $(1, 2)$, followed by the line segment from $(1, 2)$ to $(6, 7)$,
(5) followed by the line segment from $(6, 7)$ to $(13, 3)$, followed by the line segment from $(13, 3)$ to $(1, 0)$. Calculate $\int_C \cos(x)\,\vec{\imath} \cdot d\vec{r}$.