Mathematics 2443-005 Name (please print) ______________________

Examination II

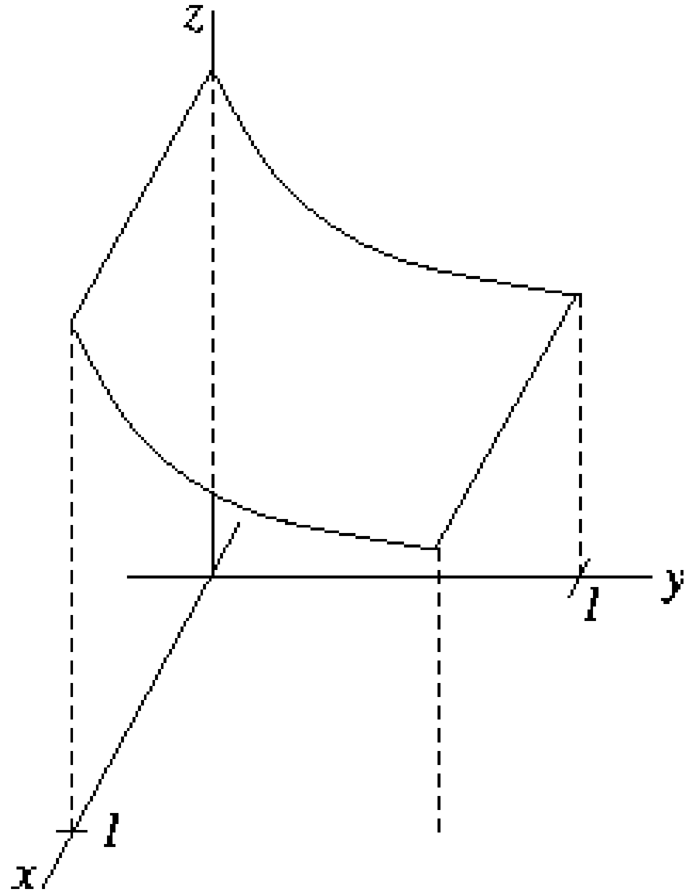
October 24, 2002

I. (4) Sketch the region of integration and change the order of integration: $\int_0^1 \int_{y^2}^{2-y} f(x,y)\, dx\, dy$

II. (3) Give a geometric explanation of why $dA = r\, dr\, d\theta$ in polar coordinates.

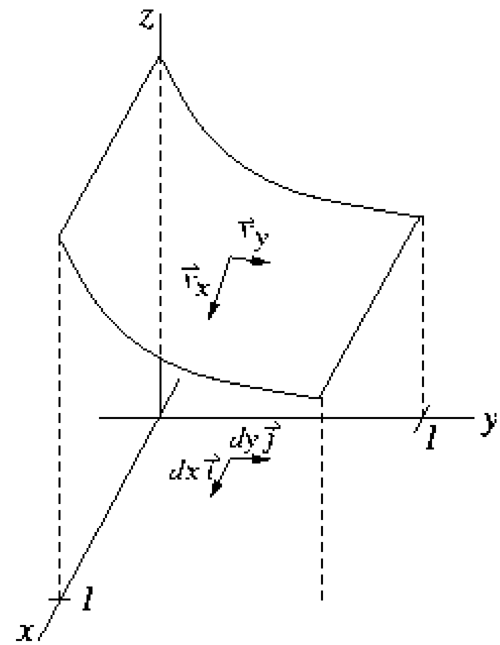
III. (6) Let T be the triangle in the first quadrant with sides $x = 0$, $y = 0$, and $x + 2y = 1$. Assume that the density at (x, y) is proportional to the distance from (x, y) to the origin. Write an integral whose value is the moment of T with respect to the y-axis (if you do not know what function to integrate, just write it as $f(x, y)$). Supply limits for integrating first with respect to x and then with respect to y, but *do not* try to calculate the value of the integral.

IV. (3) The figure to the right shows the graph of a function $f(x, y)$ whose domain is the square $0 \leq x \leq 1$ and $0 \leq y \leq 1$. In the space below, draw an xy-coordinate system and sketch the gradient of $f(x, y)$.



V. (7) Calculate $\iiint_E z\, dV$, where E lies between the spheres $\rho = 1$ and $\rho = 2$ and above the cone $\phi = \pi/4$. You may need to use some of the formulas $x = \rho\cos(\theta)\sin(\phi)$, $y = \rho\sin(\theta)\sin(\phi)$, $z = \rho\cos(\phi)$, $dV = \rho^2 \sin(\phi)\, d\rho\, d\theta\, d\phi$.

VI. (3) The figure to the right shows the graph of a function $f(x, y)$ whose domain is the square $0 \le x \le 1$ and $0 \le y \le 1$. It also shows the vectors $\vec{v}_x$ and $\vec{v}_y$ which are tangent to the surface and lie above the "vectors" $dx\ \vec{\imath}$ and $dy\ \vec{\jmath}$. Given that $\vec{v}_x = dx\ \vec{\imath} + f_x\ dx\ \vec{k}$ and $\vec{v}_y = dy\ \vec{\jmath} + f_y\ dy\ \vec{k}$, calculate $\vec{v}_x \times \vec{v}_y$.



VII. (4) Let E be the solid ball $x^2 + y^2 + z^2 \le 1$. Supply limits for the integral $\iiint_E f(x, y, z)\, dz\, dy\, dx$.

VIII. (7) Calculate the surface area of the portion of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 3$.

IX. (9) Let C be the line segment $x = 1 - t^2$, $y = t^2$, $0 \le t \le 1$.

1. Use $ds = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt$ to calculate ds. Then calculate $\int_C y^2\, ds$.

2. Calculate $\int_C y^2\, dx$.

3. Calculate $\int_C y^2 \vec{\jmath} \cdot d\vec{r}$, where $\vec{r}(t) = (1 - t^2)\vec{\imath} + t^2 \vec{\jmath}$.

X. (6) Let $\vec{F}(x, y) = \dfrac{-y}{x^2 + y^2}\,\vec{\imath} + \dfrac{x}{x^2 + y^2}\,\vec{\jmath}$.

1. Check that $\vec{F}(x, y)$ is perpendicular to the position vector of (x, y).

2. Draw the unit circle in an xy-plane, showing what the vectors $\vec{F}(x, y)$ look like at points on the circle.

3. If $\vec{F}(x, y)$ were the gradient of some function f, how would the values of f change as you travel counterclockwise around the unit circle? Why would this be impossible?