Mathematics 2443-005 Name (please print) ____________________

Examination I

September 24, 2002

I. Let $f(x, y) = x^2y^3$.
(8)

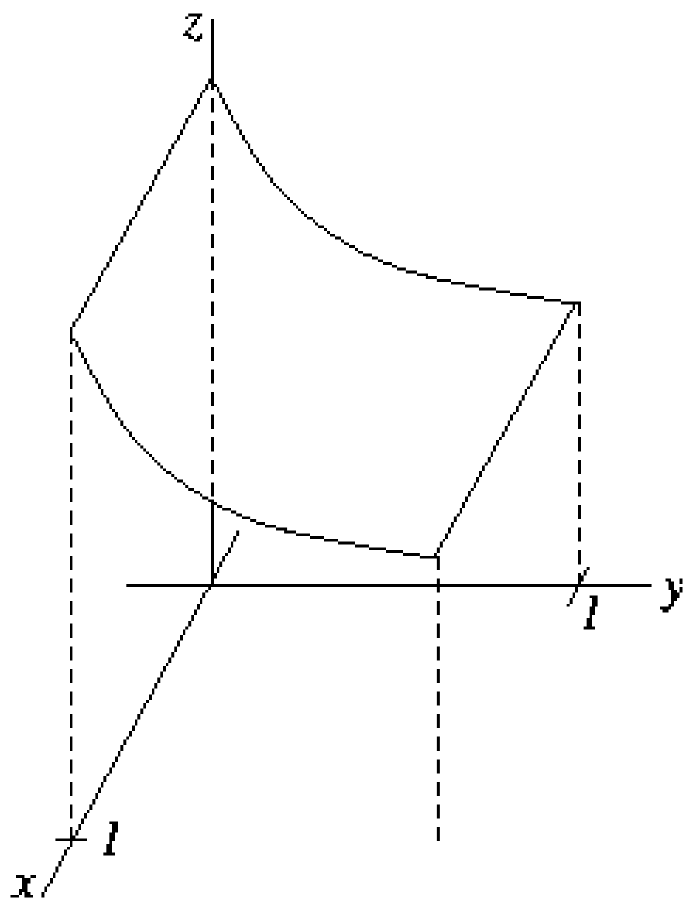
1. Calculate $\nabla f(x, y)$ and $\nabla f(-2, 1)$.

2. Find the largest directional derivative of f at the point $(-2, 1)$, and the direction in which it occurs.

3. Find the directional derivative of f at the point $(-2, 1)$, in the direction toward $(-3, 3)$.

4. Write an equation for the tangent line to the level curve of f through the point $(-2, 1)$.

II. The figure to the right shows the graph of a function $f(x, y)$ with domain the points (x, y) with $-1 \le x \le 1$ and $-1 \le y \le 1$. Answer the following questions, based on the apparent behavior of f shown on the graph.
(6)



1. At what points (x, y) is $f_x(x, y) < 0$?

2. At what points (x, y) is $f_x(x, y) > 0$?

3. At what points (x, y) is $f_y(x, y) < 0$?

4. At what points (x, y) is $f_y(x, y) > 0$?

5. At what points (x, y) is $f_{xx}(x, y) > 0$?

6. At what points (x, y) is $f_{yy}(x, y) > 0$?

III. Calculate each of the following.
(9)

1. $f_z(x, y, z)$ if $f(x, y, z) = e^{x^2 y \sin(z)}$

2. $\dfrac{\partial R}{\partial R_1}$ if $\dfrac{1}{R^3} = \dfrac{1}{R_1^2} + \dfrac{1}{R_2^2} + \dfrac{1}{R_3^2}$, using implicit differentiation.

3. $d(xy/z^2)$

IV. Calculate $(\vec{\imath} + f_x(x_0, y_0)\,\vec{k}\,) \times (\,\vec{\jmath} + f_y(x_0, y_0)\,\vec{k}\,)$. Tell how the resulting vector is related to the surface
(4) $z = f(x, y)$ at the point (x_0, y_0, z_0).

V. A closed box has length ℓ, width w, and height h.
(5)

1. Give a formula for its surface area S in terms of ℓ, w, and h.

2. Use the Chain Rule to calculate the rate at which the surface area of the box is changing when $\ell = 2$, $w = 3$, $h = 1$, $d\ell/dt = -1$, $dw/dt = 0.5$, and $dh/dt = 2$.

VI. Let $f(x, y) = xy^2$. Calculate a Riemann sum for f on the domain $0 \leq x \leq 2$, $0 \leq y \leq 4$, using the midpoint rule to select the points where the function values are computed. Use $m = n = 2$ (that is, the x-interval and y-interval are each partitioned into two subintervals of equal length).
(4)

VII. Calculate the double integral $\iint_R \dfrac{x}{1 + y^2}\, dA$, where $R = \{(x, y) \mid 0 \leq x \leq 1,\ 0 \leq y \leq 1\}$.
(4)

VIII. (4) Let $f(x,y) = \dfrac{x^2y^2}{x^2+y^2}$. Verify that $f(0,y) = 0$. For $x \neq 0$, make an estimate of $\dfrac{x^2y^2}{x^2+y^2}$ that makes it clear that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2} = 0$.

IX. (3) Verify that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist.

X. (4) The figure to the right shows the level curves of a function $f(x,y)$. Draw the gradients at the points P and Q. Indicate which one is shorter, and tell why.