Mathematics 2433-007 Name (please print) ______________________

Final Examination

December 17, 2002

I. (4) Determine whether the planes $x + 2y = 4z + 3$ and $2x + 13 = 3y + z$ meet at right angles.

II. (5) The lines given by the parametric equations $(2 - t, 5 + t, 3t)$ and $(11s - 9, s + 4, s - 1)$ meet at the point $(2, 5, 0)$. Write parametric equations for the line that passes through that point and meets both of these two lines perpendicularly.

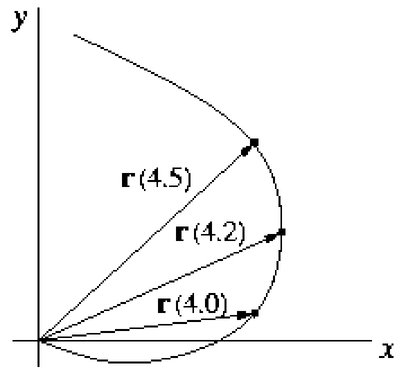
III. (6) Write parametric equations for the following curves.

1. The circle of radius 6 centered at the origin.

2. The circle of radius 6 centered at the point $(3, -7)$.

3. The portion of the graph of $y = \dfrac{2x + 2}{x^2 - 7}$ between $x = -2$ and $x = 0$.

IV. (5) The figure to the right shows a curve C given by a vector function $\vec{r}(t)$.



1. Draw the vectors $\dfrac{\vec{r}(4.5) - \vec{r}(4.0)}{0.5}$, $\dfrac{\vec{r}(4.2) - \vec{r}(4.0)}{0.2}$, and $\vec{r}\,'(4.0)$.
2. Write an expression for $\vec{r}\,'(4)$ as a limit.

V. (12) For each of the following series, tell whether the series is a geometric series, a p-series, or neither. Determine whether the series converges or diverges. Do so by applying one of the standard convergence tests, or by using your knowledge of the convergence behavior of geometric series and p-series.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1.001}}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\pi^n}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^4(n)}{n^2}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n!}$

VI. (5) Let $\sum_{n=1}^{\infty} a_n$ be a series. Give the precise mathematical definition of what it means to say that this series converges to L. Define what it means to say that the series converges absolutely.

VII. (7) This problem concerns the curve given by the vector function $\vec{r}(t) = 3\sin(t)\,\vec{\imath} + 3t\,\vec{\jmath} + 3\cos(t)\,\vec{k}$.

1. Calculate the velocity vector $\vec{r}\,'(t)$, the speed $\dfrac{ds}{dt} = \|\,\vec{r}\,'(t)\,\|$ and the arclength $s(t) = \displaystyle\int_0^t \frac{ds}{dt}\,dt$.

2. Solve for t in terms of s, and use the resulting formula to reparameterize the curve with respect to arclength (that is, write the curve as $\vec{r}(s)$).

3. For the reparameterized curve $\vec{r}(s)$, verify that $\dfrac{d\vec{r}(s)}{ds}$ has unit length.

VIII. (4) Write the Maclaurin series for $\cos(x)$ and $\ln(1+x)$, both in summation notation and by writing out the first five nonzero terms.

IX. (6) Write the general formula for the Taylor series of a function $f(x)$ at $x = a$. For the function $f(x) = \dfrac{1}{(x-1)^2}$, use the general formula to calculate the Taylor series at $a = -2$.

X. (5) This problem concerns the vector function $\vec{r}(t) = \frac{1}{3}t^3\,\vec{\imath} + t^2\,\vec{\jmath} + 2t\,\vec{k}$.

1. Calculate the velocity vector and the speed.

2. Use the formula $\kappa = \dfrac{\| \vec{T}\,'(t) \|}{\| \vec{r}\,'(t) \|}$ to calculate the curvature.

XI. (4) Let $\sum_{n=0}^{\infty} c_n (x-a)^n$ be a power series. Tell the three possible kinds of convergence behavior that are possible for this series. For each kind, be sure to indicate where the series is known to converge absolutely.

XII. (5) The following problem concerns the functions $g(x) = x^2$ and $h(x) = x^4$.

1. Use the formula $\kappa(x) = \dfrac{|f''(x)|}{(1 + f'(x)^2)^{1/2}}$ to calculate the curvature for each of the curves $y = g(x)$ and $y = h(x)$.

2. Calculate the curvatures of $y = g(x)$ and $y = h(x)$ at $x = 0$.

3. Sketch a coordinate system containing the graph of the curves near 0 (not a tiny coordinate system, please), and explain why the values of κ when $x = 0$ are consistent with the geometric behavior of the two graphs.

XIII. (5) Graph the equation $y = \frac{1}{2} + \sin(\theta)$ in a (y, θ)-plane. Determine the values of θ where that graph crosses the θ-axis. Then, graph the polar equations $r = \frac{1}{2} + \sin(\theta)$ in an xy-plane.

XIV. (4) The figure to the right shows a curve C given by a vector function. An acceleration vector $\vec{a}$ of length 3 is shown.

1. Draw the *unit* tangent and normal vectors $\vec{T}$ and $\vec{N}$.

2. Draw the tangential and normal component vectors $\vec{a}_T$ and $\vec{a}_N$ of $\vec{a}$.