Mathematics 2443-006 Name (please print) ______________________

Examination III

November 29, 2001

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**I.** Let $C$ be the circle of radius 2, centered at the origin in the $xy$-plane, and oriented counterclockwise. Use
(9) a parameterization of $C$ to give a direct calculation of each of the following line integrals.

1. $\displaystyle\int_C x^2\,ds$

2. $\displaystyle\int_C y\,dx$

3. $\displaystyle\int_C x\vec{\jmath}\cdot d\vec{r}$

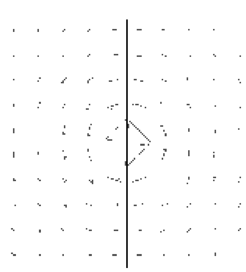
**II.** Write the Roman numeral of the vector field in front of its formula.
(4)

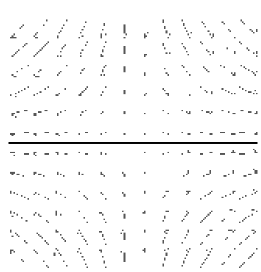
1. ______ $y\vec{\imath}+\vec{\jmath}$

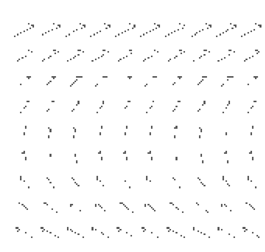
2. ______ $x\vec{\imath}+y\vec{\jmath}$

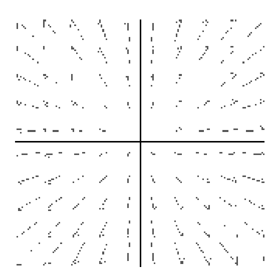
3. ______ $\dfrac{-y}{x^2+y^2}\,\vec{\imath}+\dfrac{x}{x^2+y^2}\,\vec{\jmath}$

4. ______ $x\vec{\imath}-y\vec{\jmath}$

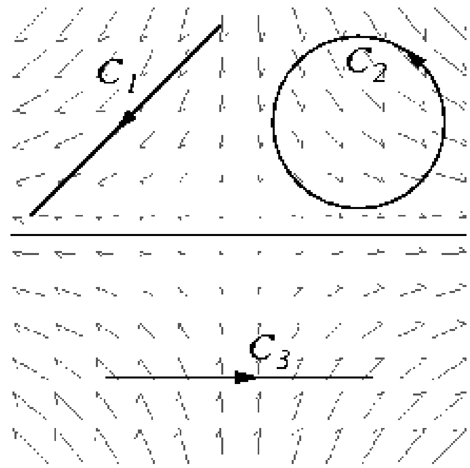
I.


II.


III.


IV.


**III.** (6) The vector field $\vec{F}$ shown to the right is conservative. Tell whether each of the following line integrals appears to be positive, negative, or zero.




1. ______ $\int_{C_1} \vec{F} \cdot d\vec{r}$

2. ______ $\int_{C_2} \vec{F} \cdot d\vec{r}$

3. ______ $\int_{C_3} \vec{F} \cdot d\vec{r}$

**IV.** (4) Answer Y or N according to whether the expression is or is not meaningful.

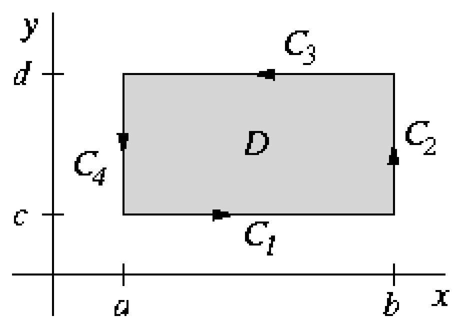
1. ______ $\operatorname{curl}(\operatorname{div}(\operatorname{curl}(\vec{F})))$

2. ______ $\nabla(\operatorname{div}(\nabla f))$

3. ______ $\operatorname{grad}(\operatorname{curl}(\vec{F}) \cdot \operatorname{grad} g)$

4. ______ $\operatorname{curl}(\operatorname{curl}(\vec{F}) \times \operatorname{grad} g)$

**V.** (6) Let $P(x,y)$ and $Q(x,y)$ be functions with continuous partial derivatives, defined on the rectangle $D$ shown to the right. The four sides of $D$ are $C_1$, $C_2$, $C_3$, and $C_4$. Verify the following.




1. $\int_{C_2} P\,dx + Q\,dy = \int_c^d Q(b,y)\,dy$ (Hint: parameterize $C_2$ as $x = b$, $y = t$, $c \le t \le d$ .)

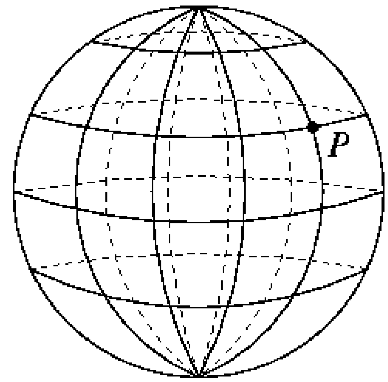
2. $\iint_D \frac{\partial Q}{\partial x}\,dA = \int_c^d Q(b,y)\,dy - \int_c^d Q(a,y)\,dy$ (Hint: supply limits of integration, integrating first with respect to $x$, then use the Fundamental Theorem of Calculus.)

**VI**. Calculate $\operatorname{curl}(f(y)\vec{\imath} + g(z)\vec{\jmath} + h(x)\vec{k})$.
(3)

**VII**. Find $f$ for which $\nabla f = (x+y)\vec{\imath} + (x+y)\vec{\jmath} + \sin(z)\vec{k}$.
(4)

**VIII**. Use Green's Theorem to calculate $\int_C \left( (e^{x^2} - 2y^3)\,\vec{\imath} + (e^{y^2} + 2x^3)\,\vec{\jmath} \right) \cdot d\vec{r}$, where $C$ is the unit circle.
(4)

**IX.** (10) The figure to the right shows the sphere $S$ of radius $a$, and a point $P$ which lies in the front part of $S$ (the coordinates of $P$ are approximately $\left(\frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}\right)$). Parameterize the sphere as $x = a\cos(\theta)\sin(\phi)$, $y = a\sin(\theta)\sin(\phi)$, $z = a\cos(\phi)$. For this parameterization, the outward normal vector is $a\sin(\phi)\,(x\vec{\imath} + y\vec{\jmath} + z\vec{k})$.




1. Draw the vectors $\vec{r}_\theta$ and $\vec{r}_\phi$ at the point $P$. Draw the outward normal vector at $P$ for this parameterization, and label it correctly as either $\vec{r}_\theta \times \vec{r}_\phi$ or $\vec{r}_\phi \times \vec{r}_\theta$.

2. Using the formula $dS = \|\,\vec{r}_u \times \vec{r}_v\,\|\,dR$, calculate $\iint_S z^2\,dS$.

3. Using the formula $\iint_S \vec{F}\cdot d\vec{S} = \iint_R \vec{F}\cdot(\vec{r}_u \times \vec{r}_v)\,dR$, calculate $\iint_S (x\vec{\imath} + y\vec{\jmath} + z\vec{k})\cdot d\vec{S}$.

**X.** (4) Bonus problem: A certain conservative vector field $\vec{F}$ is of the form $(y^3+1)^{20}\cos(x)\vec{\imath} + g(x,y)\vec{\jmath}$, for some function $g(x,y)$. Let $C$ be the upper half of the unit circle, oriented clockwise. Calculate the exact numerical value of $\int_C \vec{F}\cdot d\vec{r}$.