Mathematics 2433-003H Name (please print) ____________

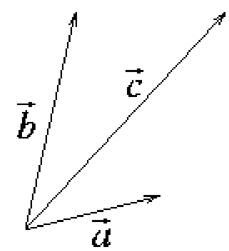
Examination III

November 27, 2001

I. (7) Let L be the line which is the intersection of the planes $2x+3y-z=7$ and $x-y+2z=1$. Give parametric equations for L.

II. (5) Let Q and R be distinct points on a line L, and let P be any point. Let $\vec{a}$ be the vector from Q to P, and $\vec{b}$ the vector from Q to R. Use the fact that $\|\vec{a}\times\vec{b}\|$ is the area of the parallelogram spanned by $\vec{a}$ and $\vec{b}$ to give a geometric proof that the distance from P to L is $\dfrac{\|\vec{a}\times\vec{b}\|}{\|\vec{b}\|}$.

III. (8) In the picture to the right, the vector $\vec{c}$ equals $\|\vec{b}\|\,\vec{a}+\|\vec{a}\|\,\vec{b}$. All three vectors lie in the plane of this piece of paper.



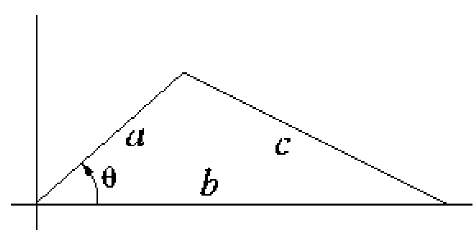
1. Use the dot product to check that $\vec{c}$ bisects the angle between $\vec{a}$ and $\vec{b}$.
2. Tell geometrically why $\vec{c}\times\vec{a}$ and $\vec{c}\times\vec{b}$ point in opposite directions.
3. Verify algebraically that $\vec{c}\times\vec{a}$ and $\vec{c}\times\vec{b}$ point in opposite directions.

IV. (6) Graph the following sets of points in a single standard xyz-coordinate system.

1. The points whose spherical coordinates satisfy $1\le\rho\le 2$, $\theta=\pi/2$, and $0\le\phi\le 3\pi/4$.
2. The points whose spherical coordinates satisfy $\rho=1$, $0\le\theta\le\pi$, and $\phi=\pi/2$.
3. The point whose spherical coordinates are $\rho=1$, $\theta=3\pi/2$, and $\phi=1.5$.

V. (8)

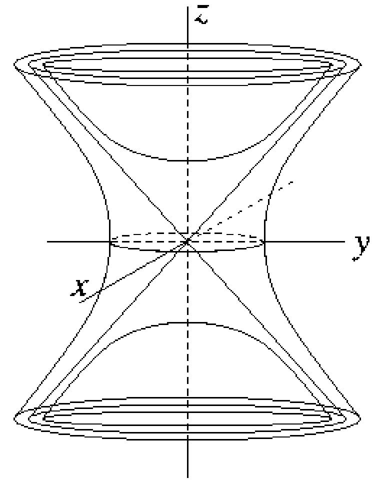
1. Use the figure to the right to prove the law of cosines, $c^2=a^2+b^2-2ab\,\cos(\theta)$.
2. If $\vec{v}=v_1\vec{\imath}+v_2\vec{\jmath}$ and $\vec{w}=w_1\vec{\imath}+w_2\vec{\jmath}$ are vectors in the plane, verify that $\vec{v}\cdot\vec{w}=\|\vec{v}\|\,\|\vec{w}\|\,\cos(\theta)$, where θ is the angle between $\vec{v}$ and $\vec{w}$.



VI. (10) Let C be the cube $0\le x\le 1$, $0\le y\le 1$, and $0\le z\le 1$.

1. Write an equation for the smallest sphere that encloses C.
2. Find the cosine of the angle between the diagonal (starting at the origin) of the face of C in the xy-plane and the diagonal of the face in the yz-plane.
3. Find the volume of the parallelepiped spanned by the diagonals of the three faces that lie in the three coordinate planes.

VII. (6) In the picture to the right, there are three surfaces (one of which has two connected pieces) which have equations of the form $Ax^2 + By^2 + Cz^2 = D$, where each of A, B, C, and D is an element of the set $\{-1, 0, 1\}$. Write the three equations, indicating which equation corresponds to which surface.



VIII. (5) Find an equation for the tangent plane to the sphere $x^2 + y^2 + z^2 = 6z$ at the point $(2, 1, 1)$.

IX. (5) Use the figure to the right to calculate x, y, and z in terms of ρ, θ, and ϕ.