Mathematics 2433-003H Name (please print) ____________________

Examination II

November 1, 2001

Instructions: Give brief, clear answers. If needed, use Taylor's Theorem, which asserts that for $f(x) - T_n(x) = R_n(x,a) = \int_a^x \frac{(x-t)^n}{n!} f^{(n+1)}(t)\, dt$ (where $T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!} (x-a)^k$), and use Lagrange's form for the remainder, $R_n(x,a) = f^{(n+1)}(c) \frac{(x-a)^{n+1}}{(n+1)!}$ for some c between a and x.

I. (6) Find a power series representation for the function $f(x) = \dfrac{x}{x+5}$ and determine its interval of convergence.

II. (6) Using the fact that $\dfrac{d}{dx}(\tan^{-1}(x)) = \dfrac{1}{1+x^2}$, find the Maclaurin series of $\tan^{-1}(x)$.

III. (6) Write the Maclaurin series of e^x. Verify that it equals e^x for all x, by showing that $\lim_{n\to\infty} R_n(x,0) = 0$ for each fixed value of x.

IV. (6) For a certain power series $\sum_{n=0}^{\infty} c_n x^n$, it is known that $\sum_{n=0}^{\infty} c_n$ converges.

1. If $\sum_{n=0}^{\infty} (-1)^n c_n$ diverges, what can be said about the radius of convergence R?
2. If $\sum_{n=0}^{\infty} (-2)^n c_n$ diverges, what can be said about the radius of convergence R?

V. (8) For the function $f(x) = x^2$, calculate the coefficients in the Taylor series $\sum_{n=0}^{\infty} c_n (x-10)^n$. Use Lagrange's form of the remainder $R_3(x, 10)$ to verify that $f(x)$ equals the value of its Taylor series for all x.

VI. (6) Find the c_n for which $\displaystyle\int \frac{1-\cos(x)}{x^2}\, dx = C + \sum_{n=0}^{\infty} c_n x^{2n+1}$.

VII. (6) Find the intersection of the sphere $(x-7)^2 + (y+8)^2 + (z-9)^2 = 64$ with each of the three coordinate planes (the xy-plane, the xz-plane, and the yz-plane).

VIII. (4) Describe in words the region in $\mathbb{R}^3$ represented by the inequality $\frac{1}{4} < x^2 + y^2 + z^2 \le 5$.

IX. (6) A certain series $\sum c_n$ converges.

1. Can the alternating series convergence criterion be applied to $\sum (-1)^n c_n$? Why or why not?
2. Suppose that $\lim_{n\to\infty} \dfrac{|c_{n+1}|}{|c_n|}$ exists and equals L. What can be said about L?

X. (6) A certain series $\sum c_n$ converges. All c_n are positive.

1. Can the alternating series convergence criterion be applied to $\sum_{n=0}^{\infty} (-1)^n c_n$? Why or why not?
2. Suppose that $\lim_{n\to\infty} \dfrac{|c_{n+1}|}{|c_n|}$ exists and equals L. What can be said about L?