Instructions: Give brief, clear answers. If needed, take as known the convergence behavior of the geometric series \( \sum r^n \) and the \( p \)-series \( \sum \frac{1}{n^p} \).

I. Use the Integral Test to check that \( \sum \frac{1}{n} \) diverges. Draw a picture showing the underlying geometric reason for the divergence.

II. State the Monotonicity Theorem for sequences. Why is it a theorem not so much about sequences as about the real numbers?

III. Use the Alternating Series Test to show that the series \( \sum (-1)^n \frac{1}{n} \) converges. Use the Ratio Test and the limit \( \lim_{x \to \infty} (1 + \frac{c}{x})^x = e^c \) to show that it converges absolutely.

IV. Use the formula \( ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \ d\theta \) to calculate \( ds \) if \( r^2 = \cos(2\theta) \). Simplify the resulting expression.

V. Give examples of:
   1. A divergent series of positive terms \( \sum a_n \) for which \( \sum a_n^2 \) is convergent.
   2. A convergent series \( \sum a_n \) such that \( \sum a_n^2 \) is divergent.

VI. Write a sum of two definite integrals in polar coordinates whose value is the area of the shaded region in the diagram to the right, but do not evaluate the integrals or proceed further with finding the area.

VII. Calculate \( \sum_{n=0}^{\infty} \cos^2n(\theta) \) (the answer is \( \csc^2(\theta) \)).

VIII. Describe the motion of a particle \( P \) that moves according to the parametric equations \( x = \cos(\cos(t)) \), \( y = \sin(\cos(t)) \).

IX. For each of the following series, determine whether the series is convergent or divergent, and give an explanation of your reasoning.
   1. \( \sum_{k=1}^{\infty} k^{-2.3} \)
   2. \( \sum_{n=1}^{\infty} a_n^2 \), given that \( \sum a_n \) is absolutely convergent (hint: for large \( n \), \( |a_n| < 1 \))
   3. \( \sum_{n=1}^{\infty} \ln(a_n) \), given that \( \sum a_n \) is a convergent series with positive terms