Mathematics 2443-003 Name (please print) ____________________

Examination III

December 1, 2000

I. (6) Let D be the domain in the plane consisting of all (x, y) with $x > 0$. It is simply-connected.

1. Use partial derivatives to verify that the vector field $\left(\dfrac{y}{x} + x\right)\vec{\imath} + (\ln(x) + y)\,\vec{\jmath}$ is conservative.

2. Find a function $f(x, y)$ with $\nabla f = \left(\dfrac{y}{x} + x\right)\vec{\imath} + (\ln(x) + y)\,\vec{\jmath}$

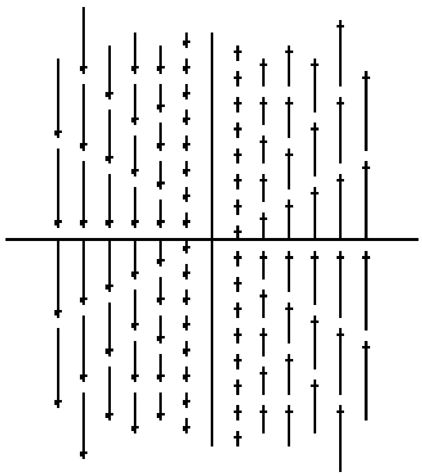
II. (6) Let R be the rectangle in the plane with vertices the points $(2, 3)$, $(5, 3)$, $(2, 4)$, and $(5, 4)$. Let C be the boundary of R, with the positive orientation. For each of the following, use a version of Green's Theorem to carry out an easy calculation of the given integral. (*Recommendation:* If this problem is taking you more than 2 minutes, go on to other problems and return to this one if you have time.)

1. $\int_C (xy^4 + y)\,dx + (2x^2y^3 - x)\,dy$.

2. $\int_C \left((2x^2y^3 + x)\vec{\imath} + (3y - xy^4)\vec{\jmath}\right) \cdot \vec{n}\; ds$, where $\vec{n}$ is the unit outward normal of C.

III. (5) Let C be the portion of the graph $y = \tan(x)$ that runs from $(\pi/4, 1)$ to $(\pi/3, \sqrt{3})$, and let $\vec{F}$ be the gradient of the function $f(x,y) = \dfrac{\ln(y)\tan x}{y}$. Calculate $\int_C \vec{F} \cdot d\vec{r}$. (*Recommendation:* If this problem is taking you more than 2 minutes, go on to other problems and return to this one if you have time.)

IV. (9) A vector field $P\vec{\imath} + Q\vec{\jmath}$ is shown to the right (on the y-axis, it consists of the zero vector at each point).



1. Tell whether $\dfrac{\partial P}{\partial x}$ is positive, negative, or zero.

2. Tell whether $\dfrac{\partial Q}{\partial x}$ is positive, negative, or zero.

3. Tell whether $\dfrac{\partial Q}{\partial y}$ is positive, negative, or zero.

4. If C is the line segment from $(0, 0)$ to $(1, 1)$, tell whether $\int_C (P\vec{\imath} + Q\vec{\jmath}) \cdot d\vec{r}$ is positve, negative, or zero.

5. If C is the line segment from $(0, 0)$ to $(1, -1)$, tell whether $\int_C (P\vec{\imath} + Q\vec{\jmath}) \cdot d\vec{r}$ is positve, negative, or zero.

6. If C is the unit circle, oriented counterclockwise, tell whether $\int_C (P\vec{\imath} + Q\vec{\jmath}) \cdot d\vec{r}$ is positve, negative, or zero.

V. (12) A curve C is parameterized as $\vec{r}(t) = \sin(t)\vec{\imath} + t^2\vec{\jmath}$ for $-1 \le t \le \pi$. Using this parameterization, express each of the following as an ordinary definite integral of a function of t, but *do not* try to calculate the actual values of the integrals.

1. $\int_C xy^2\,ds$

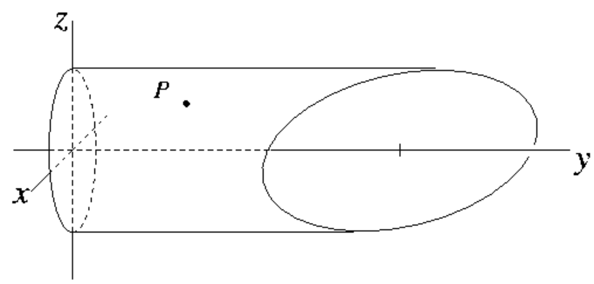
2. $\int_C xy^2\,dx$

3. $\int_C (x\vec{\imath} + y^2\vec{\jmath}) \cdot d\vec{r}$

4. For a wire bent in the shape of C, with density $\rho(x, y) = x + 3$, the moment of the wire with respect to the y-axis

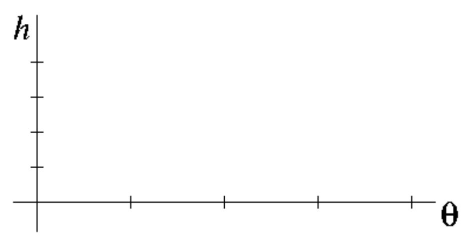
VI. (5) Calculate $\text{curl}(-y\vec{\imath} + z\vec{\jmath} - x\vec{k})$.

VII. (10) The figure to the right shows a surface S which is the portion of the cylinder $x^2 + z^2 = 1$ that lies between $y = 0$ and $x + y = 2$, and a point P which lies in the front part of S (the coordinates of P are approximately $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$). Let θ be the polar angle in the xz-plane, and let h be the distance from the xz-plane. These define parameters on S.



1. Give x, y, and z in terms of θ and h.

2. Tell the parameter domain R mathematically (that is, in terms of inequalities involving θ and h), and make an accurate drawing of it in the θh-plane shown at the right (label the coordinate axes correctly).



3. At the point P, draw and label the curve on S where θ is constant, and the curve where h is constant. Draw and label the vectors $\vec{r}_\theta$ and $\vec{r}_h$ at the point P, and draw the unit outward normal vector at P.

4. Is the unit outward normal vector $\vec{r}_\theta \times \vec{r}_h$, or is it $\vec{r}_h \times \vec{r}_\theta$?