Mathematics 2443-003 Name (please print) ______________________

Examination II

October 23, 2000

I. (10) Consider the following maximization problem: Find the volume of a rectangular box with edges parallel to the coordinate axes that can be inscribed in the ellipsoid $x^2 + \frac{y^2}{4} + z^2 = 1$.

1. Sketch the ellipsoid and the box.

2. Let (x, y, z) be the point in the first quadrant where the box meets the ellipsoid. Express the volume of the box in terms of x, y, and z.

3. Using the method of Lagrange multipliers, write four equations in the four unknowns x, y, z, and λ, so that one of the solutions of the equations corresponds to the maximum volume, but *do not* try to solve the equations or proceed further with determining the point that maximizes volume.

II. Sketch the region of integration, and change the order of integration for $\int_0^1 \int_{y^2}^{2-y} f(x, y)\, dx\, dy$.
(8)

III. Calculate each of the following integrals:
(10)

1. $\iiint_E xyz\, dV$, where E is the region in the first quadrant lying above the square $[0,1] \times [0,1]$ in the xy-plane and below the plane $z = y$.

2. $\int_0^{\pi/2} \cos^2(x)\, dx$

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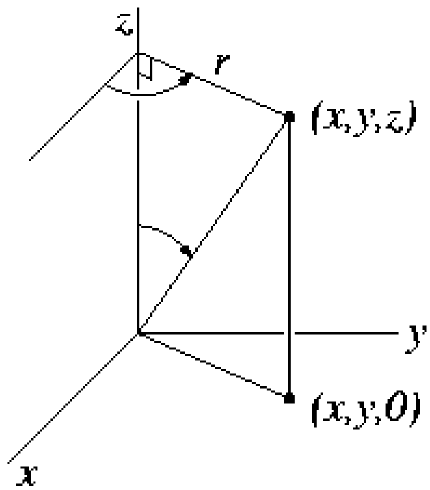
IV. Recall that in spherical coordinates, $dV = \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta$.
(12)

1. In the following picture, label ρ, θ, and ϕ, and calculate r, x, y, and z in terms of ρ, θ, and ϕ.



2. Write an integral in spherical coordinates which would calculate the mass of a rounded solid cone, consisting of the points lying above the cone $\phi = \pi/6$ and below the sphere $x^2+y^2+z^2 = 9$, if the density is proportional to the distance from the origin, with k as the constant of proportionality. Supply limits of integration, but *do not* continue with the calculation of the integral.

3. For the same solid as in the previous part, write an integral in spherical coordinates which would calculate the moment with respect to the xy-plane. Supply limits of integration, but *do not* continue with the calculation of the integral.

V. (15) This problem concerns the sphere $x^2 + y^2 + z^2 = 9a^2$ and the cylinder $x^2 + y^2 = 2ay$.

1. Graph the equation $x^2 + y^2 = 2ay$ in the xy-plane, by completing the square and putting the equation into the standard form $(x - h)^2 + (y - k)^2 = R^2$ for a circle. Also, express the equation $x^2 + y^2 = 2ay$ in polar coordinates. (If you cannot do this part of the problem, just call the region inside the circle D, and go on to the next part of the problem.)

2. Write an integral of the form $\iint_D f(x, y)\, dA$ to calculate the surface area of the portion of the sphere $x^2 + y^2 + z^2 = 9a^2$ that lies above the xy-plane and inside the cylinder $x^2 + y^2 = 2ay$.

3. Express the integral in polar coordinates. Supply limits of integration, but *do not* go on to calculate the value of the integral.