Mathematics 2443-003 Name (please print) ____________________

Examination I

September 20, 2000

I. Calculate the following partial derivatives.
(16)

1. $\dfrac{\partial w}{\partial u}$ if $w = e^{tuv}$

2. $\dfrac{\partial u}{\partial y}$ if $u = \dfrac{x^2 + y^3}{x^3 + y^2}$

3. u_{zz} if $u = \ln\sqrt{x^2 + y^2 + z^2}$

4. $\dfrac{\partial z}{\partial \theta}$ if $z = f(x, y)$

II. (9) Let $f(x, y, z) = x^3y^2z$.

1. Calculate ∇f.

2. Calculate the rate of change of f in the direction of $\vec{\imath} - \vec{\jmath}$ at the point $(-1, 1, 1)$.

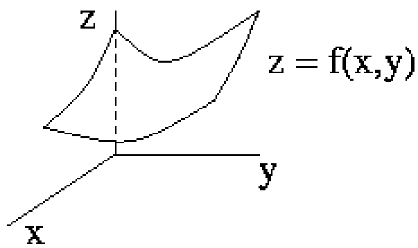
3. Tell a vector $\vec{u}$ for which $D_{\vec{u}}f(-1, 1, 1) = 0$.

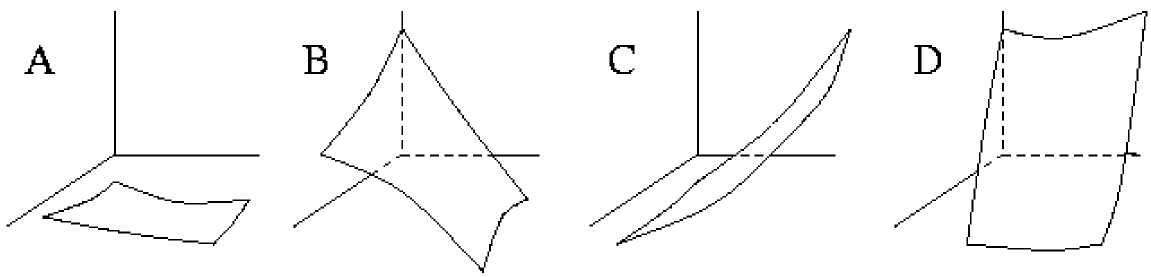
4. In what direction is f *decreasing* most rapidly at $(-1, 1, 1)$, and what is the rate of change of f in that direction?

III. (4) Write the chain rule for $\dfrac{\partial x}{\partial f}$ if $x = x(u, v)$, $u = u(f, g, h)$, and $v = v(f, g, h)$.

IV. (4) Explain why there is no function $f(x, y)$ with $f_x(x, y) = x - y^2$ and $f_y(x, y) = x^2 - y$.

V. (6) The following two questions refer to the function $f(x, y)$ whose graph is shown here, and the four functions indicated in the graphs A, B, C, and D. All of the functions have domain the points (x, y) with $0 \leq x \leq 1$ and $0 \leq y \leq 1$.





1. The graph of the function $\dfrac{\partial f}{\partial x}$ looks most like the function in

   (a) A (b) B (c) C (d) D

2. The graph of the function $\dfrac{\partial f}{\partial y}$ looks most like the function in

   (a) A (b) B (c) C (d) D

VI. (5) Suppose that $R^2 = \sin(R_1^2) + \cos(R_2^2)$.

1. Use implicit differentiation to calculate $\dfrac{\partial R}{\partial R_2}$.

2. Write an expression for the differential dR in terms of R, R_1, R_2, dR_1, and dR_2.

VII. Let D be the domain in the xy-plane consisting of all the points (x, y) for which $x \geq 0$, $y \geq 0$, and $y \leq 1-x$.
(6) Let $f(x, y) = x^2 y$.

1. Sketch the domain D.

2. Locate all critical points of f in the domain D.

3. Examine the values of f on the boundary of the domain D, and find the point where its maximum value occurs.