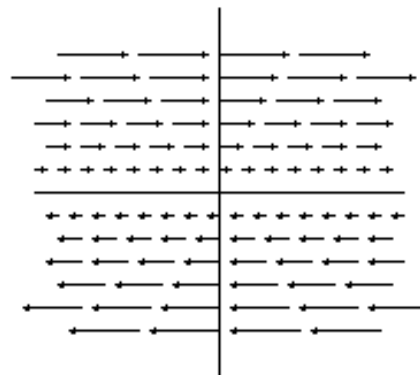


- I. Calculate $\text{curl}(xy\vec{i} + yz\vec{j} + zx\vec{k})$.
(5)

- II. A vector field $P\vec{i} + Q\vec{j}$ is shown to the right (on the y -axis, it consists of the zero vector at each point).
(4.5)



1. Tell whether $\frac{\partial P}{\partial x}$ is positive, negative, or zero.
 2. Tell whether $\frac{\partial P}{\partial y}$ is positive, negative, or zero.
 3. Tell whether $\frac{\partial Q}{\partial y}$ is positive, negative, or zero.
- III. Let C be the portion of the graph of $y = \ln(x)$ that runs from $(e, 1)$ to $(e^2, 2)$, and let \vec{F} be the gradient of the function $f(x, y) = \frac{\ln(x)y^2}{x}$. Calculate $\int_C \vec{F} \cdot d\vec{r}$. (*Recommendation:* If this problem is taking you more than 2 minutes, go on to other problems and return to this one if you have time.)
(5)

IV. A curve C is parameterized as $\vec{r}(t) = t^2\vec{i} + \sin(t)\vec{j}$ for $0 \leq t \leq \pi$. For each of the following, use the (12) parameterization to express the line integral as an ordinary definite integral of a function of t , but *do not* try to calculate the actual values of the integrals.

1. $\int_C x^2 y ds$

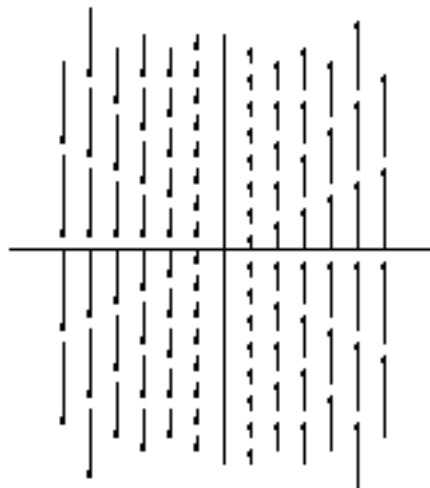
2. $\int_C x^2 y dx$

3. $\int_C (x^2\vec{i} + y\vec{j}) \cdot d\vec{r}$

4. For a wire bent in the shape of C , with density $\rho(x, y) = x + 3$, the moment of the wire with respect to the y -axis

V. The figure to the right shows a vector field (4.5) \vec{F} in the xy -plane.

1. If C is the line segment from $(1, 0)$ to $(1, 2)$, is $\int_C \vec{F} \cdot d\vec{r}$ positive, negative, or 0?
2. If C is the line segment from $(0, 1)$ to $(2, 1)$, is $\int_C \vec{F} \cdot d\vec{r}$ positive, negative, or 0?
3. If C is the unit circle, oriented counterclockwise, is $\int_C \vec{F} \cdot d\vec{r}$ positive, negative, or 0?



VI. Let D be the domain in the plane consisting of all (x, y) with $y > 0$. It is simply-connected.

(6)

1. Use partial derivatives to verify that the vector field $(\ln(y) - x)\vec{i} + \left(\frac{x}{y} - y\right)\vec{j}$ is conservative.

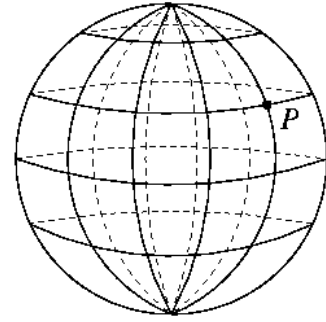
2. Find a function $f(x, y)$ with $\nabla f = (\ln(y) - x)\vec{i} + \left(\frac{x}{y} - y\right)\vec{j}$

VII. Let S be the rectangle in the plane with vertices the points $(2, 0)$, $(3, 0)$, $(2, 3)$, and $(3, 3)$. Let C be the boundary of S , with the positive orientation. For each of the following, use a version of Green's Theorem to carry out a very easy calculation of the given integral. (*Recommendation:* If this problem is taking you more than 2 minutes, work on other problems and return to this one if you have time.)

1. $\int_C (xy^4 + 3y) dx + (2x^2y^3 - 2x) dy$.

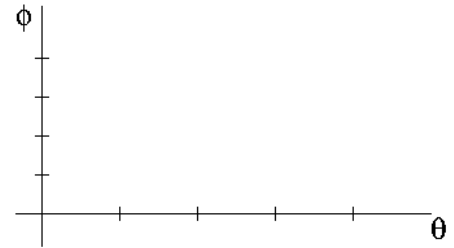
2. $\int_C ((2x^2y^3 + x)\vec{i} + (3y - xy^4)\vec{j}) \cdot \vec{n} ds$, where \vec{n} is the unit outward normal of C .

- VIII.** The figure to the right shows the sphere S of radius a , and a point P which lies in the front part of S (the coordinates of P are approximately $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$). Let ϕ and θ be the usual spherical coordinates. They define parameters on S .



1. Give x , y , and z in terms of ϕ and θ .

2. Tell the parameter domain R mathematically (that is, in terms of inequalities involving ϕ and θ), and draw it in the $\theta\phi$ -plane shown at the right (label the coordinate axes correctly).



3. At the point P , indicate the curve on S where ϕ is constant, and the curve where θ is constant. Draw and label the vectors \vec{r}_θ and \vec{r}_ϕ at the point P , and draw an outward normal vector at P .
4. Is the outward normal vector $\vec{r}_\theta \times \vec{r}_\phi$, or is it $\vec{r}_\phi \times \vec{r}_\theta$?