Mathematics 2443-002 Name (please print) ______________________

Examination I Form B

September 20, 2000

**I**. Calculate the following partial derivatives.
(16)

1. $\dfrac{\partial w}{\partial u}$ if $w = \tan^{-1}(u^2 - v^2)$

2. $\dfrac{\partial g}{\partial z}$ if $g(x, y, z) = \dfrac{x^2 + y^2 + z}{z + y - z}$

3. $u_{yy}$ if $u = \ln \sqrt{x^2 + y^2}$

4. $\dfrac{\partial z}{\partial r}$ if $z = f(x, y)$

**II.** Write the chain rule for $\dfrac{\partial w}{\partial x}$ if $w = w(f, g)$, $f = f(x, y)$, and $g = g(x, y)$.
(4)

**III.** Let $f(x, y, z) = x^3 y + \sin(z)$.
(9)

1. Calculate $\nabla f$.

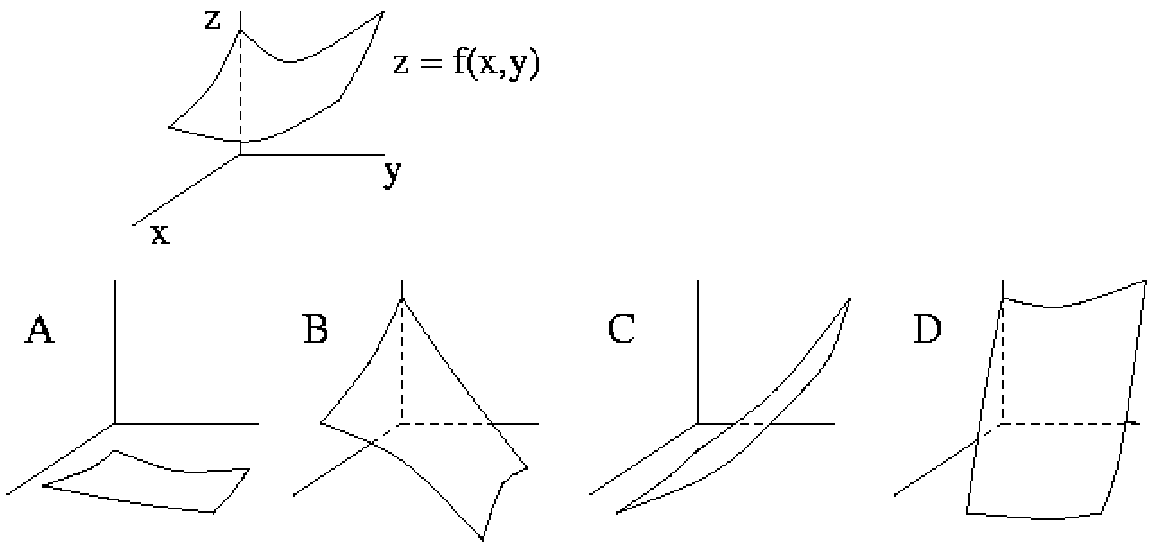
2. Calculate the rate of change of $f$ in the direction of $\vec{\imath} + \vec{\jmath}$ at the point $(-1, 1, 0)$.

3. Tell a vector $\vec{u}$ for which $D_{\vec{u}} f(-1, 1, 0) = 0$.

4. In what direction is $f$ *decreasing* most rapidly at $(-1, 1, 0)$, and what is the rate of change of $f$ in that direction?

**IV.** Explain why there is no function $f(x, y)$ with $f_x(x, y) = x - y^2$ and $f_y(x, y) = x^2 - y$.
(4)

**V.** The following two questions refer to the function $f(x, y)$ whose graph is shown here, and the four functions
(6) indicated in the graphs A, B, C, and D. All of the functions have domain the points $(x, y)$ with $0 \le x \le 1$ and $0 \le y \le 1$.




1. The graph of the function $\dfrac{\partial f}{\partial x}$ looks most like the function in

   (a) A (b) B (c) C (d) D

2. The graph of the function $\dfrac{\partial f}{\partial y}$ looks most like the function in

   (a) A (b) B (c) C (d) D

**VI.** Suppose that $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$.
(5)

1. Use implicit differentiation to calculate $\dfrac{\partial R}{\partial R_2}$.

2. Write an expression for the differential $dR$ in terms of $R$, $R_1$, $R_2$, $dR_1$, and $dR_2$.

**VII.** Let $D$ be the domain in the $xy$-plane consisting of all the points $(x, y)$ for which $x \geq 0$, $y \geq 0$, and $y \leq 1 - x$.
(6) Let $f(x, y) = x^2 y$.

1. Sketch the domain $D$.

2. Locate all critical points of $f$ in the domain $D$.

3. Examine the values of $f$ on the boundary of the domain $D$, and find the point where its maximum value occurs.