Mathematics 2443-002 Name (please print) ____________________

Examination I Form A

September 20, 2000

**I**. Calculate the following partial derivatives.
(16)

1. $\dfrac{\partial g}{\partial z}$ if $g(x,y,z) = \dfrac{x^2+y^2+z}{z+y-z}$

2. $\dfrac{\partial w}{\partial v}$ if $w = \tan^{-1}(u^2 - v^2)$

3. $u_{xx}$ if $u = \ln\sqrt{x^2+y^2}$

4. $\dfrac{\partial z}{\partial \theta}$ if $z = f(x,y)$

**II.** Explain why there is no function $f(x,y)$ with $f_x(x,y) = x - y^2$ and $f_y(x,y) = x^2 - y$.
(4)

**III.** Write the chain rule for $\dfrac{\partial z}{\partial x}$ if $z = z(f,g)$, $f = f(x,y)$, and $g = g(x,y)$.
(4)

**IV.** Let $f(x,y,z) = x^3 y + \sin(z)$.
(9)

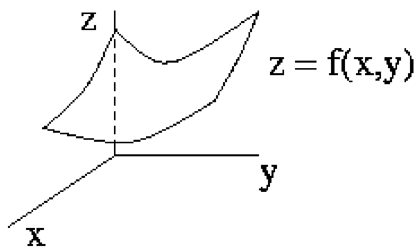
1. Calculate $\nabla f$.

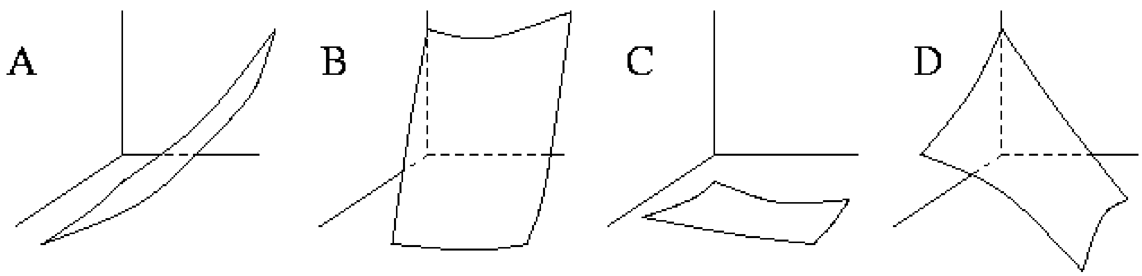
2. Calculate the rate of change of $f$ in the direction of $\vec{\imath} + \vec{\jmath}$ at the point $(1,-1,0)$.

3. Tell a vector $\vec{u}$ for which $D_{\vec{u}} f(1,-1,0) = 0$.

4. In what direction is $f$ *decreasing* most rapidly at $(1,-1,0)$, and what is the rate of change of $f$ in that direction?

**V.** (6) The following two questions refer to the function $f(x, y)$ whose graph is shown here, and the four functions indicated in the graphs A, B, C, and D. All of the functions have domain the points $(x, y)$ with $0 \le x \le 1$ and $0 \le y \le 1$.




1. The graph of the function $\dfrac{\partial f}{\partial x}$ looks most like the function in

   (a) A  (b) B  (c) C  (d) D

2. The graph of the function $\dfrac{\partial f}{\partial y}$ looks most like the function in

   (a) A  (b) B  (c) C  (d) D

**VI.** (5) Suppose that $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$.

1. Use implicit differentiation to calculate $\dfrac{\partial R}{\partial R_1}$.

2. Write an expression for the differential $dR$ in terms of $R$, $R_1$, $R_2$, $dR_1$, and $dR_2$.

**VII.** Let $D$ be the domain in the $xy$-plane consisting of all the points $(x, y)$ for which $x \geq 0$, $y \geq 0$, and $y \leq 1-x$.
(6) Let $f(x, y) = xy^2$.

1. Sketch the domain $D$.

2. Locate all critical points of $f$ in the domain $D$.

3. Examine the values of $f$ on the boundary of the domain $D$, and find the point where its maximum value occurs.