STUDY GUIDE FOR TEST 2

Chapter summary

3.8. **Endpoint problems and eigenvalues.** Understand the concept of boundary value problems, and what it means to be an eigenvalue or eigenvector. Know how to calculate eigenvalues and eigenvectors of simple boundary value problems. You do not need to learn any of the application problems in this chapter. These problems typically involve a lot of trigonometry, so make sure you understand all that very well (e.g., you can tell for which α $\cos(4\alpha) = 0$ or $\tan(\alpha\pi) = 0$) *Example problems: 3.8 1-9,11-14*

5.1. **Matrices and linear systems.** Know how to converta simple linear system into matrix form. Understand the principle of superposition for homogeneous systems, linear independence, and the difference between general and specific solutions. Know how to calculate the Wronskian for 2 and 3-vectors. *Example problems: 5.1 11-40.*

7.1. **Laplace transforms and inverse transforms.** Understand how to calculate Laplace transforms using the definition. Understand what it means that Laplace transforms are linear. Understand how to take inverse Laplace transforms. Know the condition for a Laplace transform to exist. Note: we did not cover the $u_a(t)$ function in the book, so there is no need to know how to find Laplace transforms of this type. *Example problems: 7.1 1-6,11-35*

7.2. **Transformation of IVPs.** Understand the derivative and integral formulas. Very important: make sure you understand how to solve IVPs using Laplace transforms. Know how to use the derivative formula to calculate Laplace transforms, for example using the product rule. *Example problems: 7.2 1-31*

7.3. **Translation and Partial Fractions.** Make sure you understand partial fractions from calculus very well. Learn to recognize when it would be useful to use partial fractions to break up fractions in inverse Laplace transform problems. Understand how to use partial fractions to solve IVPs. *Example problems: 7.3 1-38*

7.4. **Derivatives, Integrals and Products of Transforms.** Understand how to take a convolution product of two functions (this involves knowing partial fractions really well, and understanding to distinguish between the variable of integration, and variables you should treat as constants). Know how to use the convolution product to take inverse Laplace transforms, and know how to use convolution to calculate IVPs (as in problems 36-38). Understand Theorems 2 and 3, and recognize when they would be useful. *Example problems 7.4 15-34.*

Notes on Chapter 7

A lot of the problems in Chapter 7 can also be solved by the methods of Chapter 3. For the third midterm, I will insist that you use the methods of Chapter 7 to solve IVPs (i.e., please don't use the substitution $x(t) = e^{rt}$, etc.). **Using the methods of Chapter 3 will get you a 0 on the problem even if you do it perfectly correct**. Also, a lot of inverse Laplace transforms can be calculated by multiple methods. Make sure you know all the methods. I reserve the right to insist on a certain method for solving a problem.

There are a number of derivations in Chapter 7, a lot of which are easy and would make good test questions. Here is a comprehensive list of derivations that you would be expected to know: every Laplace transform in Figure 7.1.2 other than $u(t-a)$, t^n for $n \geq 3$ and t^a for a non-integer. The proof of linearity of Laplace transforms (Theorem 1 of 7.1). The s-translation formula (Theorem 1 of 7.3). The transforms of derivatives formula (Theorem 1 of 7.2, only for continuous functions). Most of the preceding derivations are also on my YouTube channel.

This is the formula page that will be on your test, possibly with minor modifications (e.g. there might be blanks on the Laplace transform table if I want to ask you to derive one of those Laplace transforms)

$f(t)$	$\mathcal{L}(f(t))$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$t^a(a > -1)$	$\frac{\Gamma(a+1)}{s^{a+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin(kt)$	$\frac{s}{s^2+k^2}$
$\cos(kt)$	$\frac{k}{s^2+k^2}$
$\sinh(kt)$	$\frac{k}{s^2-k^2}$
$\cosh(kt)$	$\frac{s}{s^2-k^2}$

The above Laplace transforms are valid for $s > 0$, with the exception of $\sinh(kt)$ and $\cosh(kt)$ which are valid for $s > |k|$.

$$n \text{ integer}, \Gamma(n+1) = n!$$

$$\mathcal{L}(f'(t)) = s\mathcal{L}(f(t)) - f(0)$$

$$\mathcal{L}(f^{(n)}(t)) = s^n\mathcal{L}(f(t)) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left(\int_0^t f(\theta)d\theta\right) = \frac{\mathcal{L}(f(t))}{s}$$

$$F(s) = \mathcal{L}(f(t)) \implies F(s-a) = \mathcal{L}(e^{at}f(t))$$

$$f(t) * g(t) = \int_0^t f(\theta)g(t-\theta)d\theta$$

$$\mathcal{L}(-tf(t)) = \frac{dF(s)}{ds}$$

$$\mathcal{L}\left(\frac{f(t)}{t}\right) = \int_0^s F(\sigma)d\sigma$$