# Practice Midterm Exam II
# Math 101 Summer General Session 2010

Instructions: This is a **105**-minute exam. You may not consult any notes or books during the exam, and no calculators are allowed. Show all of your work on each problem. Attach extra paper if you need more space.

Write your name:

Write out the Honor Pledge: "On my honor, I have neither given nor received any unauthorized aid on this exam."

Signature:

| Problem | Score |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| Total | |

## Some useful identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\pi \text{ radians} = 180^\circ$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$\log(ab) = \log a + \log b$$

$$\log(a^b) = b\log a$$

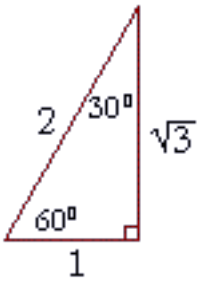
$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

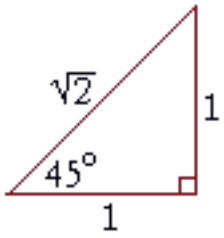
$$\ln(x) = \log_e(x)$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$




You may use the above triangles to recall certain values of $\sin, \cos$, and $\tan$.

1. (10 points)

   (a) Calculate

   $$\sum_{i=1}^{5}(i+1).$$

   (b) Express

   $$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$$

   in $\Sigma$-notation.

2. (10 points) Use the technique of linear approximation to estimate the following quantities:

   (a) $\sqrt{82}$.

   (b) the change in the area of a square, if its edge length decreases from 10cm to 9.8cm.

3. (5 points) Joe is driving along a highway with speed limit 70 miles per hour. He passes through two toll booths that are thirty miles apart. He reaches the first toll booth at 2:10pm, and the second toll booth at 2:35pm. Prove that at some point, Joe was speeding.

4. (15 points) Find $f'(x)$, $f''(x)$, and $f'''(x)$ when

   (a) $f(x) = \sin(3x)$.

   (b) $f(x) = x^3 + x^2$.

   (c) $f(x) = e^{-x}$.

5. (15 points) Make a detailed sketch of the graph $y = \frac{\ln x}{x}$ on $(0, \infty)$. Make sure that you clearly note

   (a) all inflection points and all maxima/minima.

   (b) where the graph is increasing,decreasing, concave up or concave down.

   (c) the horizontal and vertical asymptotes. There is one of each. Write down the equations for those asymptotes.

6. (10 points)Use l'Hôpital's rule to find the following limits:

(a)

$$\lim_{x\to\infty} \frac{x^2}{e^x}.$$

(b)

$$\lim_{x\to 0^+} x^x.$$

7. (10 points) Find the following antiderivatives:

(a)

$$\int x(x+1)dx.$$

(b)

$$\int \sin(x) - 2\cos(x)dx.$$

8. (10 points) Calculate the following definite integrals:

(a)

$$\int_0^1 (2x+1)^5 dx.$$

(b)

$$\int_{-1}^2 \frac{x+1}{(x^2+2x-1)^2}.dx$$

9. (5 points) A penny dropped from the top of a building lands on the pavement three seconds later. Acceleration due to gravity is 32 feet per second per second. How tall is the building?

10. (5 points) Find the area of the planar region bounded below by the line $y = 16$ and bounded above by the parabola $y = 25 - x^2$.

11. (5 points) Estimate $\int_0^{\pi} f(x)dx$ using Simpson's Approximation with $n = 4$, where $f(x) = \frac{\sin(x)}{x}$ when $x$ nonzero, and $f(0) = 1$.