Practice Final Exam
Math 101 Summer General Session 2010

Instructions: This is a **105**-minute exam. You may not consult any notes or books during the exam, and no calculators are allowed. Show all of your work on each problem. Attach extra paper if you need more space.

Write your name:

Write out the Honor Pledge: "On my honor, I have neither given nor received any unauthorized aid on this exam."

Signature:

Problem	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Some useful identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\pi \text{ radians} = 180^\circ$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$\log(ab) = \log a + \log b$$

$$\log(a^b) = b\log a$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\ln(x) = \log_e(x)$$

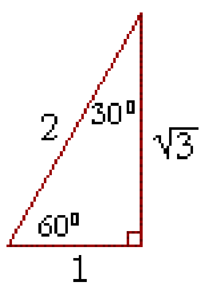
$$\log_a(b) = \frac{\ln b}{\ln a}$$

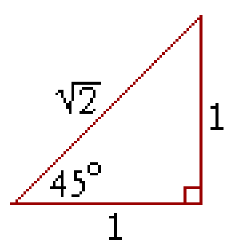
$$a^b = e^{b\ln a}.$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$





Useful Formulas

The volume of a planar region under the curve $y = f(x)$ rotated around the x-axis is given by

$$\int_a^b \pi f(x)^2 dx.$$

The volume of a planar region under the curve $y = f(x)$ rotated around the y-axis is given by

$$\int_a^b 2\pi x f(x) dx.$$

The length of the curve $y = f(x)$ from $x = a$ to $x = b$ is

$$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx.$$

The surface area of a planar region under the curve $y = f(x)$ rotated around the x-axis is given by

$$\int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx.$$

The surface area of a planar region under the curve $y = f(x)$ rotated around the y-axis is given by

$$\int_a^b 2\pi x \sqrt{1 + [f'(x)]^2} dx.$$

The coordinates of the centroid $(\bar{x}, \bar{y})$ of a planar region under the curve $y = f(x)$ with area A is given by

$$\bar{x} = \frac{1}{A} \int_a^b x f(x) dx,$$

$$\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} f(x)^2 dx.$$

1. (20 points) Find $\frac{dy}{dx}$ when

   (a) $y = \pi$.

   (b) $y = e^{-x} \sin x$.

   (c) $y = \frac{x}{\cos x}$

   (d) $y = 10^{1/x}$.

2. (20 points) Evaluate the following integrals:

   (a) $y = \int_0^1 x dx$.

   (b) $y = \int \frac{\sin x}{\cos x} dx$.

   (c) $y = \int 3^{2x} dx$.

   (d) $y = \int \frac{1}{\sqrt{4+x^2}} dx$.

3. (10 points)Use l'Hôpital's rule to find the following limits:

(a)

$$\lim_{x\to\infty} \frac{\ln x}{e^x}.$$

(b)

$$\lim_{x\to 0} x^{\sin x}.$$

4. (5 points) The width of a rectangle is half its length. At what rate is its area increasing if its width is 10cm and is increasing at a rate of 0.5 cm per second?

5. (5 points) Prove that the function $x^7 + x^5 + x^3 + 1 = 0$ has exactly one real solution.

6. (10 points) A particle is moving along a line with velocity function $v(t) = 2t + 10$. From the time $t = 1$ to the time $t = 5$, calculate the particle's

   (a) net distance traveled

   (b) total distance traveled.

7. (10 points) Calculate the following volumes.

(a) The region bounded by $y = x^2$ and $x = y^2$, rotated around the x-axis.

(b) The region bounded by $y = x^2, y = 0, x = 1, x = -1$, rotated around the line $x = 2$.

8. (10 points)

(a) Set up and simplify the integral that gives the surface area of revolution generated by rotation of the arc $y = x^2$, for $0 \leq x \leq 4$ around the y-axis. **Do not evaluate the integral!**

(b) Find the length of the smooth arc $y = \frac{e^x + e^{-x}}{2}$ from $x = 0$ to $x = 1$.

9. (10 points) Find the centroid of the planar region bounded by $y = x^2, y = 18 - x^2$.