Homework # 7: 5463: Real Analysis 2

1. Compute the Fourier transforms of

$$f(x) = \chi_{(-1,1)}(x), \quad g(x) = \chi_{(-1,1)}(x)(1-|x|).$$

2. Suppose that $f \in L^1(\mathbb{R})$ is bounded and continuous, and

$$\left|\widehat{f}(\xi)\right| \leq \frac{C}{1+|\xi|^{1+\alpha}},$$

for some $0 < \alpha < 1$. Show that then f is *Hölder continuous* of order α, that is, there exists a constant M so that

$$|f(x+h) - f(x)| \leq M|h|^{\alpha}$$

for all $x, h \in \mathbb{R}$.

Hint: Use the Fourier inversion formula to write down the difference $f(x+h) - f(x)$. Then treat separately the contributions coming from $|\xi| \leq 1/|h|$ and $|\xi| > 1/|h|$.

Remark: These two problems illustrate a general principle: Smoothness properties of the function correspond to decay properties of the Fourier transform.

3. Exercises 8, 9 from Section 8.2.

Hint: In Exercise 8, please make sure you know exactly what you want to show. Then the actual verification is not that complicated (an application of Hölder's inequality).

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