Homework # 6: 5463: Real Analysis 2

1. Let $\{r_n\}$ be an enumeration of $\mathbb{Q} \cap (0,1)$, let $g(x) = \chi_{(0,1)}(x)\, x^{-1/2}$, and define

$$f(x) = \sum_{n=1}^{\infty} 2^{-n} g(x - r_n)$$

(compare Exercise 25 from Section 2.3).

(a) Prove the following facts about f again: The series defining f converges for a.e. $x \in (0,1)$, and f is measurable (in fact, $f \in L^1(0,1)$).
Also, recall (but don't prove) that f is unbounded on every open set; in particular, f is discontinuous at every point.

(b) Therefore, Lusin's Theorem can be applied, and we in particular obtain the following: For every $\epsilon > 0$, there exists a set $E \subset (0,1)$ with $m(E^c) < \epsilon$ so that the restriction of f to E is continuous. Construct such a set explicitly.

2. Exercises 1, 2, 3 from Section 8.1.

"Due" 3/13