Homework # 2: 5463: Real Analysis 2

1. For each of the following (measurable) functions, determine all $p \geq 1$ for which $f \in L^p(\mathbb{R})$:

$$f_1(x) = \sin x$$
$$f_2(x) = x$$
$$f_3(x) = \chi_{\mathbb{Q}}(x)$$
$$f_4(x) = \chi_{(1,\infty)}(x)\,\frac{1}{x}$$
$$f_5(x) = \chi_{(0,1)}(x)\,\frac{1}{x^{1/2}}$$
$$f_6(x) = \frac{1}{x}$$

2. (a) What are the sets of measure zero for the counting measure on $\mathbb{N}$? Deduce from your answer that on $\ell^\infty = L^\infty(\mathbb{N}, \mu)$,

$$\|a\|_\infty = \sup_{n\in\mathbb{N}} |a_n|.$$

(b) Let $a \in \ell^1$ (that is, $\sum_{n\geq 1} |a_n| < \infty$). Then $a \in \ell^p$ for all $1 \leq p \leq \infty$ by Proposition 6.11. Prove that

$$\|a\|_\infty = \lim_{p\to\infty} \|a\|_p.$$

(Compare Exercise 6.1.7.)

3. Exercises 5, 11 from Section 6.1

Discussion: 1/31