Homework solution # 1: Complex Analysis 1

1. *Proof.* Assume first that A is closed. Let $z_n \in A$, $z \in \mathbb{C}$, $z_n \to z$. We must show that $z \in A$. This is actually clear because if we had $z \notin A$, then, since $\mathbb{C} \setminus A$ is open, we could find a whole disk $D(z, \epsilon) = \{w \in \mathbb{C} : |w - z| < \epsilon\}$ with $D(z, \epsilon) \cap A = \emptyset$. However, $z_n \to z$ implies that $z_n \in D(z, \epsilon)$ for all large n, and this contradicts our assumption that $z_n \in A$.

   Conversely, assume the condition on the sequences and let $z \in \mathbb{C} \setminus A$. We must find a disk $D(z, \epsilon) \subseteq \mathbb{C} \setminus A$. If that is not possible, then for each n we can choose a $z_n \in D(z, 1/n)$, $z_n \in A$. Clearly, $z_n \to z$, so $z \in A$ by hypothesis. We have again reached a contradiction. □

2. *Proof.* Pick a point $z_n \in K_n$ from each set. Then in particular $z_n \in K_1$ for all n, and this set is compact, so we can extract a convergent subsequence $z_{n_k} \to z$. Since K_1 is closed, we obtain that $z \in K_1$. Now $z_n \in K_j$ for $n \geq j$ and any fixed j and again this set is closed, so it also follows that $z \in K_j$, and thus z is a point in the intersection. □