Linear Algebra

Name:__

1. Let
$$S = \left\{ \begin{bmatrix} 1\\0 \end{bmatrix}, \begin{bmatrix} 0\\1 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{2}\\1/\sqrt{2} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{2}\\-1/\sqrt{2} \end{bmatrix} \right\}.$$

(a) Find the lengths of the vectors in S. (2 pts)

(b) Is the set S orthogonal, orthonormal, or neither? Explain. (4 pts)

2. Find an orthogonal basis for the 2-dimensional subspace of \mathbb{R}^3 with basis $\left\{ \begin{bmatrix} -2\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\2\\3 \end{bmatrix} \right\}$. (6 pts)

3. Let $S = {\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}}$ be an orthonormal set of vectors in \mathbb{R}^n . The following two questions are multiple choice - circle the best possible answer. You do not need to show work or explain your answer. (4 points each)

Question 1: What can we say about n?

- (a) n = 3
- (b) $n \le 3$
- (c) $n \ge 3$
- (d) $n \neq 3$
- (e) We cannot tell anything about n from the given information

Question 2: Let $\mathbf{u} = \mathbf{v_1} - \mathbf{v_3}$ and $\mathbf{w} = 3\mathbf{v_1} + 2\mathbf{v_2}$. What is $\mathbf{u} \cdot \mathbf{w}$?

- (a) 1
- (b) 3
- (c) 4
- (d) 9
- (e) This cannot be determined from the given information