Quiz 1 Solutions

1. The following matrices are augmented matrices of linear systems which are in REF or RREF. Determine the number of solutions to each system and find all solutions. Write the solutions as vectors. (5 pts each)

   (a) $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

   If we label the variables as x, y, z, this system corresponds to the equations $x + 2y - z = 4, y + z = 3, z = 1, 0 = 0$. Plugging in $z = 1$ to the second equation gives $y = 2$. Plugging in $z = 1$ and $y = 2$ to the first equation gives $x = 1$. So there is one solution, it is $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

   (b) $\left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 2 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$

   Label the variables x, y, z, w. There is no leading one in the z column so z can be anything and the other variables can be solved for in terms of z. The equations are $x + 2y + 3z = 2, y + z = 1, w = 4$. From this, we have that $w = 4$, z is anything, $y = 1 - z$, and $x = 2 - 2y - 3z = 2 - 2(1 - z) - 3z = -z$. There are infinite solutions. They are any vectors of the form $\begin{bmatrix} -z \\ 1 - z \\ z \\ 4 \end{bmatrix}$ where z is any number.

   (c) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

   This system has no solutions as the last row corresponds to the equation $0 = 1$.

2. Let A be an $m \times n$ matrix and $\mathbf{0}$ be the zero m-vector. If $\mathbf{v}$ is a solution to $A\mathbf{x} = \mathbf{0}$, is $5\mathbf{v}$ a solution to $A\mathbf{x} = \mathbf{0}$? Why or why not? (5 pts)

   Yes. As $\mathbf{v}$ is a solution to $A\mathbf{x} = \mathbf{0}$, this means that $A\mathbf{v} = \mathbf{0}$. Then $A(5\mathbf{v}) = 5(A\mathbf{v}) = 5\mathbf{0} = \mathbf{0}$ so $5\mathbf{v}$ is also a solution to $A\mathbf{x} = \mathbf{0}$.