Homework 9 Due: Tues, April 7

Book Problems:
Section 4.8 # 3, 5, 11, 15, 27
Section 6.1 # 1, 11b, 12, 15, 16

Additional Problems:

1. Let S be the following ordered basis for $\mathbb{R}^3$. $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$.

   (a) Let $\mathbf{v} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}$. Find $[\mathbf{v}]_S$, the coordinate vector of $\mathbf{v}$ with respect to S.

   (b) Suppose $\mathbf{w}$ is a vector in $\mathbb{R}^3$ and $[\mathbf{w}]_S = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Find $\mathbf{w}$.

2. Let $S = \{1, t, t^2\}$ and $T = \{t - 1, t^2 + 1, t\}$. These are both ordered bases for P_2. Let $p(t) = 4t^2 - 5t + 3$.

   (a) Find $[p(t)]_S$ and $[p(t)]_T$.

   (b) Find $P_{S \leftarrow T}$, the transition matrix from T to S.

   (c) Find $Q_{T \leftarrow S}$, the transition matrix from S to T.

   (d) Verify that $[p(t)]_S = P_{S \leftarrow T}[p(t)]_T$ and $[p(t)]_T = Q_{T \leftarrow S}[p(t)]_S$.

   (e) How are $P_{S \leftarrow T}$ and $Q_{T \leftarrow S}$ related? Hint: multiply them together.

3. For each of the following functions, determine if it is a linear transformation.

   (a) $L : \mathbb{R}^3 \to \mathbb{R}^2$ by $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} xy \\ z \end{bmatrix}$

   (b) $L : M_{23} \to M_{32}$ by $L(A) = A^T$

4. Let $L : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation $L\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + 2z \\ y - w \\ 3w + z + x \end{bmatrix}$. Find the standard matrix representing L.