Homework 8 Due: Tues, March 31

Book Problems:
Section 5.1 # 7, 17, 25
Section 5.3 # 29, 40
Section 5.4 # 1, 10, 13, 28

Additional Problems:

1. For each of the following set, determine if it is orthogonal, orthonormal, or neither.

   (a) $\left\{ \begin{bmatrix} 2/7 \\ 6/7 \\ -3/7 \end{bmatrix}, \begin{bmatrix} 9/\sqrt{146} \\ 1/\sqrt{146} \\ 8/\sqrt{146} \end{bmatrix} \right\}$

   (b) $\left\{ \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}, \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix}, \begin{bmatrix} -1/2 \\ -1/2 \\ -1/2 \\ -1/2 \end{bmatrix} \right\}$

   (c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \right\}$

2. Verify that the set $S = \left\{ \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} 2/\sqrt{6} \\ -1/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix}, \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} \right\}$ is an orthonormal basis for $\mathbb{R}^3$. Use dot products to write the vector $\begin{bmatrix} 7 \\ -2 \\ 1 \end{bmatrix}$ as a linear combination of the vectors in S.

3. Let $\mathbf{v} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$. Let W be the set of all vectors in $\mathbb{R}^3$ which are orthogonal to $\mathbf{v}$. Show that W is a subspace of $\mathbb{R}^3$. Find a basis for W and the dimension of W.

4. Let $S = \{\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_k}\}$ be a set of vectors in $\mathbb{R}^n$. Suppose that $\mathbf{u}$ is a vector in $\mathbb{R}^n$ which is orthogonal to every vector in S. Is $\mathbf{u}$ orthogonal to every vector in span S? Why or why not?

5. Find an orthonormal basis for the 3-dimensional subspace of $\mathbb{R}^4$ with basis $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$.