

Book Problems:

Section 4.6 # 13, 28b

Section 4.7 # 11, 25

Section 4.9 # 5, 35, 46

Additional Problems:

1. Let W be the subspace of \mathbb{R}^4 spanned by $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -5 \\ -1 \end{bmatrix} \right\}$. Find a basis for W and $\dim W$.

2. Let V be a finite dimensional vector space and let W be a subspace of V . Prove that if $\dim W = \dim V$, then $W = V$.

3. Let $A = \begin{bmatrix} 0 & 0 & 6 & 0 & 19 & 11 \\ 3 & 12 & 9 & -6 & 26 & 31 \\ 1 & 4 & 3 & -2 & 10 & 9 \\ -1 & -4 & -4 & 2 & -13 & -11 \end{bmatrix}$. The RREF of A is $\begin{bmatrix} 1 & 4 & 0 & -2 & 0 & 4 \\ 0 & 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

- (a) Find the rank and the nullity of A .
- (b) Find a basis for the row space of A .
- (c) Find a basis for the column space of A .
- (d) Find a basis for the null space of A .
4. Let A be a 5×9 matrix.
- (a) Find all possible values for the rank of A .
- (b) Find all possible values for the nullity of A .
- (c) If the rows of A are linearly independent, what is the rank of A ?
- (d) Are the columns of A linearly independent?
- (e) How many solutions does $A\mathbf{x} = \mathbf{0}$ have?