Homework 6 Due: Tues, March 3

Book Problems:
Section 4.5 # 3, 13, 20, 24
Section 4.6 # 1, 9, 16, 19, 33

Additional Problems:

1. Determine if the set $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 1 \\ 1 \end{bmatrix} \right\}$ is linearly independent. If it is not linearly independent, write one of the vectors in $S$ as a linear combination of the other vectors in $S$.

2. Let $V$ be a vector space and let $S = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ be a linearly independent set of vectors in $V$. Determine if the following sets are linearly independent.
   (a) $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$
   (b) $\{\mathbf{v_1}, \mathbf{v_1} + \mathbf{v_2}, \mathbf{v_1} + \mathbf{v_3}, \mathbf{v_4} - \mathbf{v_2}\}$
   (c) $\{\mathbf{v_1} + \mathbf{v_2} + \mathbf{v_3}, \mathbf{v_2} - \mathbf{v_3}, \mathbf{v_1} + 2\mathbf{v_2}, \mathbf{v_4}\}$

3. Let $S = \{\mathbf{v_1}, \mathbf{v_2}\}$ be a set of nonzero vectors in a vector space $V$. Suppose $\mathbf{w}$ is a vector in $V$ and that $\mathbf{w} = 5\mathbf{v_1} - \mathbf{v_2}$ and $\mathbf{w} = 3\mathbf{v_1} - 2\mathbf{v_2}$ (so $\mathbf{w}$ can be written as a linear combination of the vectors in $S$ in more than one way).
   (a) Prove that $S$ is not linearly independent.
   (b) What can you say about the dimension of span $S$?

4. Find a basis for and the dimension of each of the following subspaces.
   (a) The subspace of $\mathbb{R}^5$ which consists of vectors of the form $\begin{bmatrix} 4t+s \\ t-s \\ t \\ 3s \\ s \end{bmatrix}$ where $s, t$ are any real numbers.
   (b) The subspace of $M_{23}$ which consists of matrices of the form $\begin{bmatrix} a-b & c-d & 3a \\ -b & 0 & d-c \end{bmatrix}$ where $a, b, c, d$ are any real numbers.
   (c) The subspace of $P_2$ which consists of polynomials $at^2+bt+c$ with $a+b+c=0$.