Homework 5 Due: Tues, February 24

Book Problems:
Section 4.2 # 1, 11, 17
Section 4.3 # 4, 5, 17, 25
Section 4.4 # 2, 5, 9

Additional Problems:

1. Let V be the set of real numbers and define the operations $\oplus$ and $\odot$ to be the following.

   $\mathbf{u} \oplus \mathbf{v} = \mathbf{u} + \mathbf{v} - 3$ for $\mathbf{u}, \mathbf{v}$ in V
   $r \odot \mathbf{u} = r(\mathbf{u} - 3) + 3$ for $\mathbf{u}$ in V and r a real number.

   Prove that V with the operations $\oplus$ and $\odot$ is a real vector space.

2. Determine which of the following are subspaces. You may assume the operations are the usual addition and scalar multiplication in $\mathbb{R}^n$ and P.

   (a) Let V be the set of 2-vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ with $|y| = |x|$. Is V a subspace of $\mathbb{R}^2$?

   (b) Let V be the set of polynomials $p(t)$ such that $\int_0^1 p(t)\, dt = 0$. Is V a subspace of P?

   (c) Let V be the set of polynomials $p(t)$ such that $p(0) = 5$. Is V a subspace of P?

   (d) Let A be a fixed 3×3 matrix. Let V be the set of 3-vectors $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$ is a consistent linear system. Is V a subspace of $\mathbb{R}^3$?

3. Let $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} \right\}$. Does S span $\mathbb{R}^3$? Either prove that S spans $\mathbb{R}^3$, or find a vector in $\mathbb{R}^3$ which is not in the span of S.

4. Let W be the set of 3×3 skew symmetric matrices. Find a set S of 3×3 matrices such that $W = \text{span } S$. Is W a subspace of M_{33}?